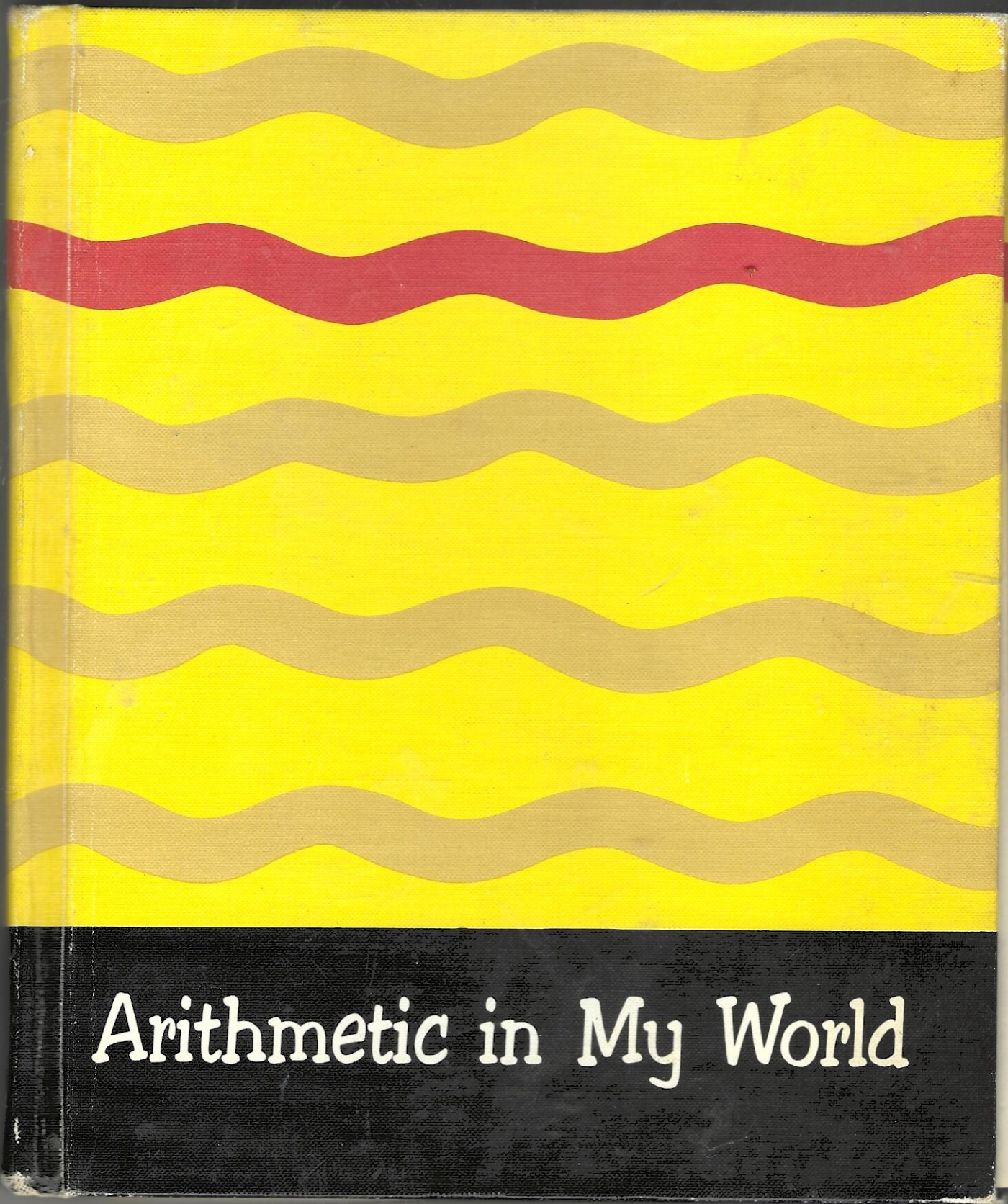

Arithmetic in My World

# Arithmetic

## IN MY WORLD

by

C. Newton Stokes
Belle Adams
Arthur R. DeLong

6



1958

ALLYN AND BACON, INC.

Boston New York Chicago Atlanta Dallas San Francisco

# ARITHMETIC IN MY WORLD

## 6

ILLUSTRATORS

Edwina Chin
Ruth Clift
Lee Dummer
Ruth Potter
Herbert Townsend
Winnifred Farnum Westlake

*The authors believe that the learning and teaching of arithmetic is based on child growth. Through the study of many thousands of children and their problems, they have determined how and what children should learn in order to establish desirable growth patterns. They have found that children's problems can be organized into units of work composed of activities which are in reality areas of development.*

*It should be noted that there are two headings for each unit in the Contents. The first heading names the social situation which the authors have established for each unit of work. The second heading indicates the associated arithmetical concepts which are developed in the unit.*

© Copyright, 1958, by Allyn and Bacon, Inc. Printed in the United States of America.

# Contents


6 Solving Everyday Problems . . 168
Addition and Subtraction of Fractions

7 Using Your Spare Time . . . . 206
Multiplication and Division of Fractions

8 Using One Number System . . . 244
Decimals

9 Learning to Use Per cent . . . 280
Practical Uses of Percentage

10 Seeing Quantity All Around You 294
Larger Values—The Four Fundamental Processes

11 Considering the Shapes and Sizes of Things . . . . . 312
Practical Geometry

TO BE
CONTINUED

# UNIT 6

# Solving Everyday Problems



Every day in your life, you have a great number of problems to solve. Here are some of them:

sharing a grapefruit or bottle of milk at breakfast.
dividing things up when you play games.
sharing the cost of something with someone else.
deciding how much cloth to buy when making a dress.
finding how much credit to give a pitcher who was knocked out of the box during an inning.
finding how much time you have to play after school.

Can you add other items to this list of times when you would need to think of wholes and parts of wholes?

Look at the boys and girls in the picture on the opposite page. Do you think they are gaining good arithmetic experience from what they are doing? Do you think that they needed previous experience working with numbers to do what they are doing? Will they be using whole units and parts of whole units?

What is the name that is sometimes given to a part of a whole?

What is a fraction?

Do these boys and girls have any use for fractions? Name them.

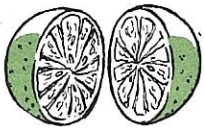
When do you need fractions in measurements? in music? in cooking? in sharing?

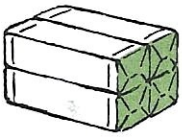
Tell the importance you see in the use of fractions in your everyday life. Where do you get experience in using fractions?

What fractions might you find in making a batch of cookies?

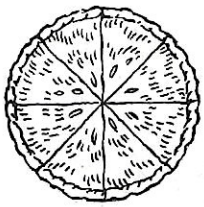
Do you ever use fractions to tell the time of day?

How do you use fractions to build a table, a bookcase, an aquarium, a birdhouse, or a rabbit pen?









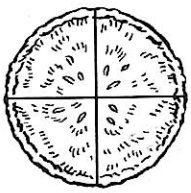
## Using Parts of Wholes

Do you like to eat grapefruit for breakfast? Do you know what the size of each part of a grapefruit will be if you and your mother share one equally? Have you ever opened a pound of butter? Was it divided into parts? What was the size of each part? How would you cut an apple in order to share it equally with two friends?

What is the size of each of the pieces of pie in the picture? How many people will that pie serve if each one gets 1 piece?

What do you mean when you say that you have ridden your bike $\frac{1}{2}$ mile? What does the 2 in this number $\frac{1}{2}$ tell you? the 1?

If a pie is cut into 8 equal parts, why can you say that $\frac{1}{8}$ is the size of 1 of the parts? Can you call the $\frac{1}{8}$ unit a measure? What would it mean if you said that you had eaten $\frac{3}{8}$ of the pie? Can this be a measurement? Why?





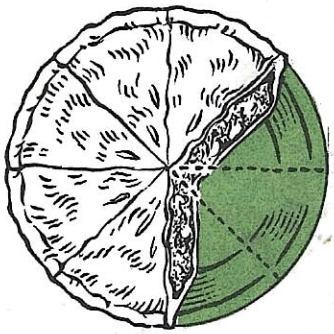
Your mother had this amount of pie left for supper. Why can you say that there is $\frac{5}{4}$ pie altogether?

There are $\frac{8}{8}$ in a whole. $\frac{3}{8}$ is only a part of the whole. Why? When there is only a part of a whole, it is called a fractional part of it.

Do you remember that: **a**. The number which expresses the part of a whole is called a proper fraction; and **b**. When you have enough parts to make a whole or more than a whole, the number that expresses how much you have is called an improper fraction?

$\frac{1}{2}$ and $\frac{3}{8}$ are proper fractions. $\frac{8}{8}$ and $\frac{5}{4}$ are improper fractions.

## The Terms of a Fraction



This pie has been divided into 8 equal parts. You know that the size of one of the parts is called one-eighth, and the fraction for this number name is $\frac{1}{8}$.

In the picture, there are 5 of the eighth units or $\frac{5}{8}$ of the pie remaining. Why?

There are two numbers in every fraction. Do you remember the names of the places which they hold?

Which number in a fraction is the denominator? What does this number tell you?

Which number in a fraction is the numerator? Is it correct to say that this number tells the how-many of the parts of some whole we have in use?

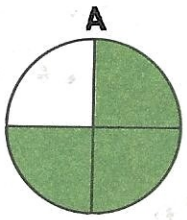
Sometimes we speak of the numerator and denominator of a fraction as the **terms of the fraction.**

Which term of a proper fraction contains the larger number? Why?

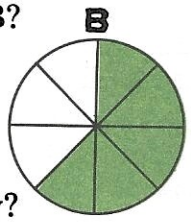
Which term is the larger in an improper fraction? Why?

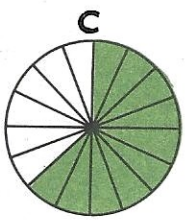
**1.** Tell which are denominators and which are numerators in each of the following fractions:

$\frac{3}{4}$ $\frac{1}{2}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{2}{9}$ $\frac{3}{10}$ $\frac{1}{7}$ $\frac{2}{3}$ $\frac{5}{12}$



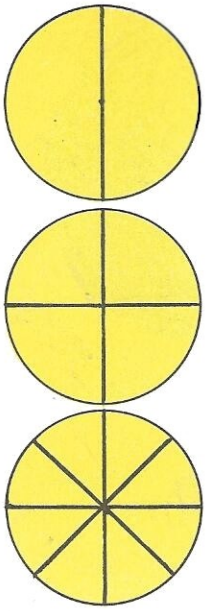

**2.** What number would you put in the denominator of a fraction to tell the part of drawing A that is shaded? of drawing B? of drawing C?




**3.** Write the fractions which tell the parts of the whole circles that are shaded.

**4.** Which is larger, $\frac{1}{4}$ or $\frac{1}{8}$ of a whole? Why?



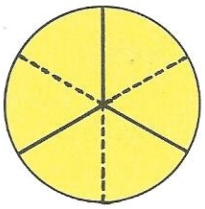
## Comparing Fractions

**1.** All the circles on the left are the same size. Tell how each one has been divided into parts.

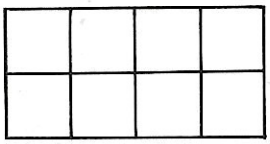
**2.** If the wholes are the same size, which is larger, $\frac{1}{2}$ of one, or $\frac{1}{4}$ of another? Why? $\frac{1}{2}$ of one, or $\frac{1}{8}$ of another? Why?

**3.** Does the denominator of a fraction help you to decide which is larger, $\frac{1}{4}$ or $\frac{1}{8}$? How?

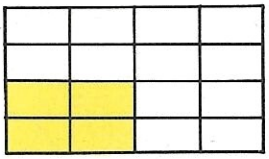
**4.** Tell which of these fractional parts is the largest part in each group: $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$; $\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{6}$; $\frac{1}{8}$ $\frac{1}{6}$ $\frac{1}{4}$



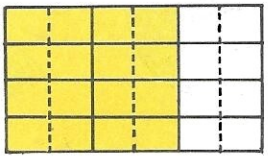
**5.** The two fractions $\frac{5}{6}$ and $\frac{1}{2}$ are each a part of a whole. Which is the larger? How can you tell? Is there any difference in the size of $\frac{1}{3}$ or $\frac{2}{6}$ of this circle? Tell why.



**6.** Use this drawing to tell why $\frac{1}{4}$ of a whole is equal to $\frac{2}{8}$ of it. Tell why $\frac{1}{2}$ of it is equal to $\frac{4}{8}$ of it.



**7.** How many different fractions can you write to tell the part of this drawing that is shaded?

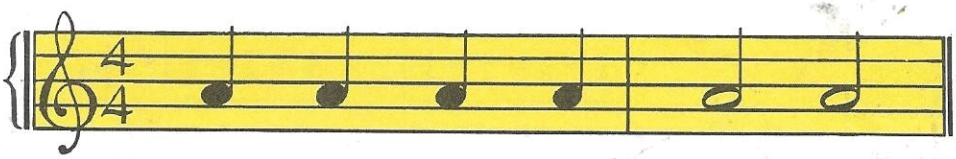


**8.** Which of these fractions are equal to $\frac{2}{3}$? (Use drawings.)

$\frac{4}{6}$ $\frac{9}{12}$ $\frac{16}{24}$ $\frac{11}{12}$ $\frac{8}{12}$ $\frac{15}{24}$

## Pete's Music Lesson

Pete was taking his first lesson in reading music. The teacher first showed him a sample. It looked like this:



He was told the first symbol represented a treble clef. The $\frac{4}{4}$ means that there are 4 quarter $(\frac{1}{4})$ notes to the measure. The four $\frac{1}{4}$'s make one whole note. A vertical line is used to show the end of a measure.

Here is a key to the value of the notes:

whole note  half note  quarter note  eighth note 

You can see that the first measure contains 4 of the $\frac{1}{4}$ notes, which equal one whole note.

The next measure has 2 of the $\frac{1}{2}$ notes to make the whole measure. Do you know how many $\frac{1}{8}$ notes it takes to make a whole measure? to make a whole note?

What is the difference between a whole note and a half note? between a quarter note and an eighth note?

The larger the note the longer the sound that it represents is played.

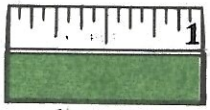
Why is a $\frac{1}{2}$ note played twice as long as a $\frac{1}{4}$ note? four times as long as a $\frac{1}{8}$ note?

Do you know what $\frac{2}{4}$ time would be? It means that in each measure there are 2 quarter notes or some other notes that are equal to 2 quarter notes. Can you name any other notes that are equal to 2 quarter notes?

Why is the number that tells that there are four $\frac{1}{4}$ notes to a measure $(\frac{4}{4})$ an improper fraction?

## Practice Exercises

In measuring with units like 1 inch, the fractional parts most commonly used are halves, fourths, eighths, and sixteenths.

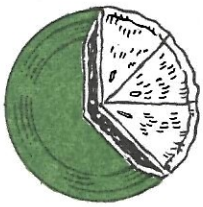


1 inch

$\frac{8}{16} = \frac{4}{8} = \frac{2}{4} = \frac{1}{2}$ $\quad$ $\frac{4}{16} = \frac{2}{8} = \frac{1}{4}$ $\quad$ $\frac{2}{16} = \frac{1}{8}$

If you start with the first fraction in each box and change to those on its right, you get larger units. Why? This is also called changing to **lower terms**. It is changing to larger units. Why?

> Remember that changing to lower terms is changing to smaller numbers in the fraction and does not change the value of the fraction.



**1.** Which of the following can be changed to larger units (answers must come out even)?

**a.** Mary gave Johnny $\frac{3}{8}$ of a pie.
**b.** Mary bought $\frac{4}{8}$ of a yard of ribbon.
**c.** Alice gave Suzy $\frac{8}{16}$ of her candy bar.
**d.** Joe's dog ate $\frac{2}{7}$ of the bones.

**2.** Change these fractions as indicated, making drawings if you need them: $\frac{1}{2} = \frac{?}{4}$ $\quad$ $\frac{1}{2} = \frac{?}{8}$ $\quad$ $\frac{1}{2} = \frac{?}{16}$ $\quad$ $\frac{1}{4} = \frac{?}{8}$ $\quad$ $\frac{1}{4} = \frac{?}{16}$

**3.** Change these to the largest unit or part possible (this means change to lowest terms):

$\frac{3}{9}$ $\quad$ $\frac{6}{8}$ $\quad$ $\frac{4}{16}$ $\quad$ $\frac{2}{8}$ $\quad$ $\frac{6}{9}$ $\quad$ $\frac{5}{10}$ $\quad$ $\frac{3}{12}$ $\quad$ $\frac{9}{12}$ $\quad$ $\frac{2}{16}$ $\quad$ $\frac{4}{8}$ $\quad$ $\frac{4}{12}$

$\frac{4}{10}$ $\quad$ $\frac{4}{6}$ $\quad$ $\frac{8}{12}$ $\quad$ $\frac{6}{10}$ $\quad$ $\frac{10}{16}$ $\quad$ $\frac{5}{15}$ $\quad$ $\frac{12}{20}$ $\quad$ $\frac{12}{16}$ $\quad$ $\frac{9}{15}$ $\quad$ $\frac{16}{20}$ $\quad$ $\frac{9}{12}$ $\quad$ $\frac{18}{29}$ $\quad$ $\frac{33}{32}$

**4. a.** Which of these fractions have a value of $\frac{1}{2}$?

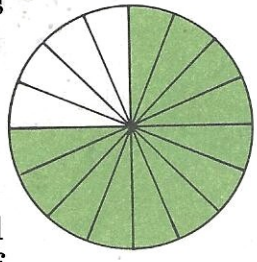
$\frac{2}{4}$ $\quad$ $\frac{6}{12}$ $\quad$ $\frac{4}{8}$ $\quad$ $\frac{6}{10}$ $\quad$ $\frac{12}{24}$ $\quad$ $\frac{12}{16}$ $\quad$ $\frac{9}{18}$ $\quad$ $\frac{10}{14}$ $\quad$ $\frac{10}{20}$ $\quad$ $\frac{4}{6}$

**b.** Find the fractions below which have the value $\frac{1}{4}$:

$\frac{2}{8}$ $\quad$ $\frac{8}{12}$ $\quad$ $\frac{3}{12}$ $\quad$ $\frac{5}{20}$ $\quad$ $\frac{4}{16}$ $\quad$ $\frac{6}{16}$ $\quad$ $\frac{6}{24}$ $\quad$ $\frac{8}{32}$ $\quad$ $\frac{12}{48}$ $\quad$ $\frac{7}{28}$

## Learning How to Change Terms

You should know that the terms of a fraction are its numerator and denominator. You should also know how to change these terms of a fraction while still keeping its value.



The part or the unit in this picture is the sixteenth. There are 12 of them shaded. Thus, $\frac{12}{16}$ of the whole are being considered.

If you can make the size of the part larger, there will be fewer parts in the whole. Why? Can you put 2 of the sixteenths together for a larger part?

In finding the size of the new part, $2\overline{)16}$ tells you the how-many of them there will be in the whole. There will be 8 parts, so the new part or unit is the eighth.

You must put 2 of the 12 parts in $\frac{12}{16}$ together, also: $2\overline{)12} = 6$. This does not change the value of the fraction. Why?

The units have been made twice as large and the number of them half as many.

Do you see that both the numerator and the denominator have been divided by the same number?

You could have put 4 of the sixteenths together for larger parts:

$$\frac{12}{16} = \frac{4\overline{)12}}{4\overline{)16}} = \frac{3}{4}.$$ Why?

The fraction $\frac{12}{16}$ is now equal to $\frac{3}{4}$. The largest number of parts has been put together to form a new part or unit. The fraction has been changed to its **lowest terms.**

If you make the parts or units smaller, there will be more of them. For example, $\frac{2}{3} = \frac{6}{9}$, where each part has been made into 3 smaller ones. So, there are 3 times as many parts in both the numerator and the denominator. Since the terms or numbers in the fraction are larger, the fraction is said to have been raised to **higher terms.**

## Practice in Changing the Terms of a Fraction

Now that you know what can be done to both the numerator and denominator of a fraction to change its value, see if you agree with the following statements:

**a.** If the numerator and denominator of a fraction are both divided evenly by the same number, the terms of the fraction are reduced. If they are divided by the largest number possible, the fraction is changed to its lowest terms.

$$\frac{16}{24} = \frac{2\overline{)16}}{2\overline{)24}} = \frac{8}{12}$$ changed to lower terms

$$\frac{16}{24} = \frac{8\overline{)16}}{8\overline{)24}} = \frac{2}{3}$$ changed to lowest terms

**b.** If the numerator and denominator of a fraction are both multiplied by the same number, the terms of the fraction are raised.

$$\frac{2}{5} = \frac{4 \times 2}{4 \times 5} = \frac{8}{20}$$ changed to higher terms

Is it possible to have a fraction raised to highest terms? Why, or why not?

Change these to lowest terms:

| | a. | b. | c. | d. | e. | f. | g. | h. |
|---|---|---|---|---|---|---|---|---|
| 1. | $\frac{9}{21}$ | $\frac{10}{25}$ | $\frac{8}{12}$ | $\frac{6}{18}$ | $\frac{16}{32}$ | $\frac{36}{48}$ | $\frac{28}{49}$ | $\frac{14}{35}$ |
| 2. | $\frac{4}{12}$ | $\frac{12}{16}$ | $\frac{15}{24}$ | $\frac{6}{20}$ | $\frac{10}{15}$ | $\frac{10}{30}$ | $\frac{4}{16}$ | $\frac{4}{10}$ |

**3.** Tell the number by which you would divide the terms of each of these fractions when changing to lowest terms. (Example: $\frac{36}{45} = \frac{12}{15} = \frac{4}{5}$. You have really divided by $3 \times 3$, or 9.)

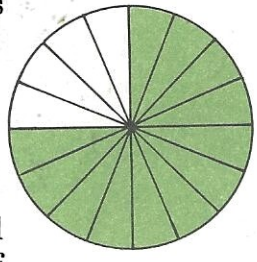
| a. | b. | c. | d. | e. | f. | g. | h. |
|---|---|---|---|---|---|---|---|
| $\frac{32}{48}$ | $\frac{30}{42}$ | $\frac{18}{54}$ | $\frac{16}{48}$ | $\frac{16}{40}$ | $\frac{20}{100}$ | $\frac{15}{75}$ | $\frac{24}{72}$ |

**4.** Change to higher terms:

| a. | b. | c. | d. | e. |
|---|---|---|---|---|
| $\frac{3}{5} = \frac{?}{10}$ | $\frac{5}{8} = \frac{?}{16}$ | $\frac{3}{7} = \frac{6}{?}$ | $\frac{5}{9} = \frac{10}{?}$ | $\frac{5}{4} = \frac{?}{8}$ |

## Learning How to Change Terms

You should know that the terms of a fraction are its numerator and denominator. You should also know how to change these terms of a fraction while still keeping its value.



The part or the unit in this picture is the sixteenth. There are 12 of them shaded. Thus, $\frac{12}{16}$ of the whole are being considered.

If you can make the size of the part larger, there will be fewer parts in the whole. Why? Can you put 2 of the sixteenths together for a larger part?

In finding the size of the new part, $2\overline{)16}$ tells you the how-many of them there will be in the whole. There will be 8 parts, so the new part or unit is the eighth.

You must put 2 of the 12 parts in $\frac{12}{16}$ together, also: $2\overline{)12} = 6$. This does not change the value of the fraction. Why?

The units have been made twice as large and the number of them half as many.

Do you see that both the numerator and the denominator have been divided by the same number?

You could have put 4 of the sixteenths together for larger parts:

$$\frac{12}{16} = \frac{4\overline{)12}}{4\overline{)16}} = \frac{3}{4}.$$ Why?

The fraction $\frac{12}{16}$ is now equal to $\frac{3}{4}$. The largest number of parts has been put together to form a new part or unit. The fraction has been changed to its **lowest terms**.

If you make the parts or units smaller, there will be more of them. For example, $\frac{2}{3} = \frac{6}{9}$, where each part has been made into 3 smaller ones. So, there are 3 times as many parts in both the numerator and the denominator. Since the terms or numbers in the fraction are larger, the fraction is said to have been raised to **higher terms**.

## Practice in Changing the Terms of a Fraction

Now that you know what can be done to both the numerator and denominator of a fraction to change its value, see if you agree with the following statements:

**a.** If the numerator and denominator of a fraction are both divided evenly by the same number, the terms of the fraction are reduced. If they are divided by the largest number possible, the fraction is changed to its lowest terms.

$$\frac{16}{24} = \frac{2\overline{)16}}{2\overline{)24}} = \frac{8}{12}$$ changed to lower terms

$$\frac{16}{24} = \frac{8\overline{)16}}{8\overline{)24}} = \frac{2}{3}$$ changed to lowest terms

**b.** If the numerator and denominator of a fraction are both multiplied by the same number, the terms of the fraction are raised.

$$\frac{2}{5} = \frac{4 \times 2}{4 \times 5} = \frac{8}{20}$$ changed to higher terms

Is it possible to have a fraction raised to highest terms? Why, or why not?

Change these to lowest terms:

| | a. | b. | c. | d. | e. | f. | g. | h. |
|---|---|---|---|---|---|---|---|---|
| **1.** | $\frac{9}{21}$ | $\frac{10}{25}$ | $\frac{8}{12}$ | $\frac{6}{18}$ | $\frac{16}{32}$ | $\frac{36}{48}$ | $\frac{28}{49}$ | $\frac{14}{35}$ |
| **2.** | $\frac{4}{12}$ | $\frac{12}{16}$ | $\frac{15}{24}$ | $\frac{6}{20}$ | $\frac{10}{15}$ | $\frac{10}{30}$ | $\frac{4}{16}$ | $\frac{4}{10}$ |

**3.** Tell the number by which you would divide the terms of each of these fractions when changing to lowest terms. (Example: $\frac{36}{45} = \frac{12}{15} = \frac{4}{5}$. You have really divided by $3 \times 3$, or 9.)

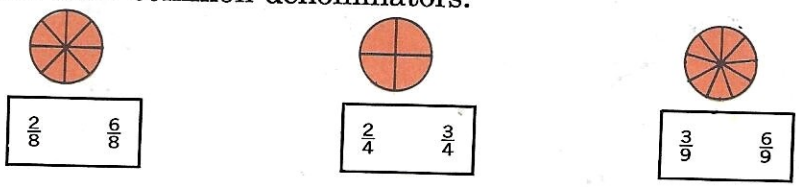
| a. | b. | c. | d. | e. | f. | g. | h. |
|---|---|---|---|---|---|---|---|
| $\frac{32}{48}$ | $\frac{30}{42}$ | $\frac{18}{54}$ | $\frac{16}{48}$ | $\frac{16}{40}$ | $\frac{20}{100}$ | $\frac{15}{75}$ | $\frac{24}{72}$ |

**4.** Change to higher terms:

| a. | b. | c. | d. | e. |
|---|---|---|---|---|
| $\frac{3}{5} = \frac{?}{10}$ | $\frac{5}{8} = \frac{?}{16}$ | $\frac{3}{7} = \frac{6}{?}$ | $\frac{5}{9} = \frac{10}{?}$ | $\frac{5}{4} = \frac{?}{8}$ |

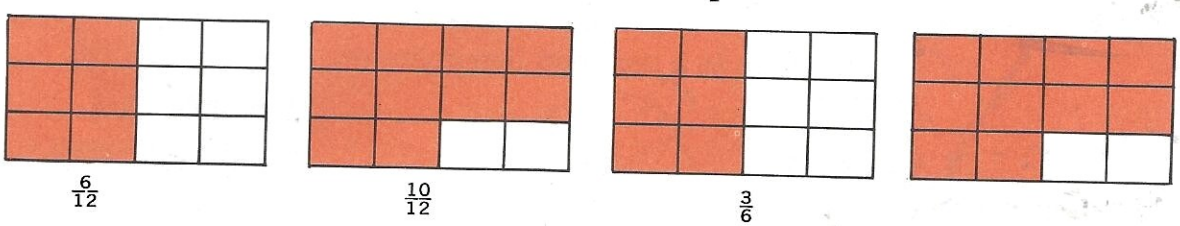
## Common Denominators

Fractions which have the same number for their denominators (like fractions) are said to have a common denominator. Does this mean that you have the same size units? Why? Such fractions as these in the different boxes have common denominators:

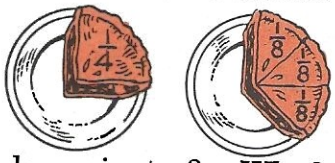



When like fractions have the smallest possible number as a denominator in each fraction, this number is called the **lowest common denominator** (L.C.D.).

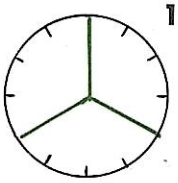
Examine these drawings and the shaded parts:




1. Why do $\frac{6}{12}$ and $\frac{10}{12}$ have a common denominator?
2. Does $\frac{6}{12} = \frac{3}{6}$ of a whole? Why?
3. Does $\frac{10}{12} = \frac{5}{6}$ of a whole? Why?
4. What do you do to change $\frac{6}{12}$ of a whole to $\frac{3}{6}$ of it?
5. Do the fractions $\frac{3}{6}$ and $\frac{5}{6}$ have a lowest common denominator? What is it?
6. How would you change the fractions $\frac{1}{2}$ and $\frac{5}{6}$ so that they would have a common denominator of 12? a lowest common denominator of 6?
7. To tell how much pie you have altogether in the drawings, would you have to change the fractional parts to a common denominator? Why? Would it have to be a lowest common denominator? Why?

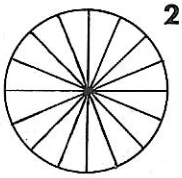


## Practice in Changing Fractions



**1.** Changing to smaller parts or units:

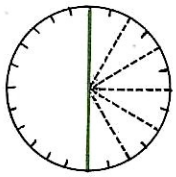
Change $\frac{2}{3}$ to sixths.
Change $\frac{1}{3}$ to twelfths.
Change $\frac{1}{6}$ to twelfths.
Are you changing to higher or lower terms?

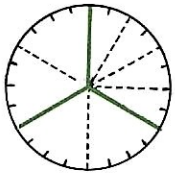


**2.** Changing to larger parts or units:

Change $\frac{8}{16}$ to halves.
Change $\frac{12}{16}$ to fourths.
Change $\frac{6}{16}$ to eighths.
Are you changing them to higher or lower terms?

**3.** Change to a common denominator:





Example: $\frac{1}{2}, \frac{1}{3}$

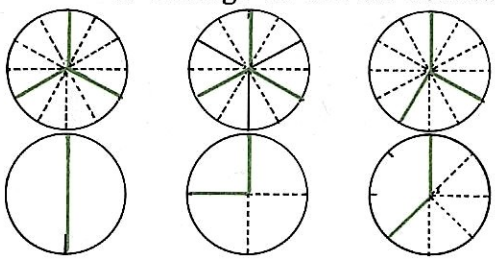
$\frac{1}{2} = \frac{3}{6} = \frac{6}{12} = \frac{12}{24}$

$\frac{1}{3} = \frac{2}{6} = \frac{4}{12} = \frac{8}{24}$

The diagrams show that we can have sixths, twelfths, or twenty-fourths. Could we also have eighteenths?

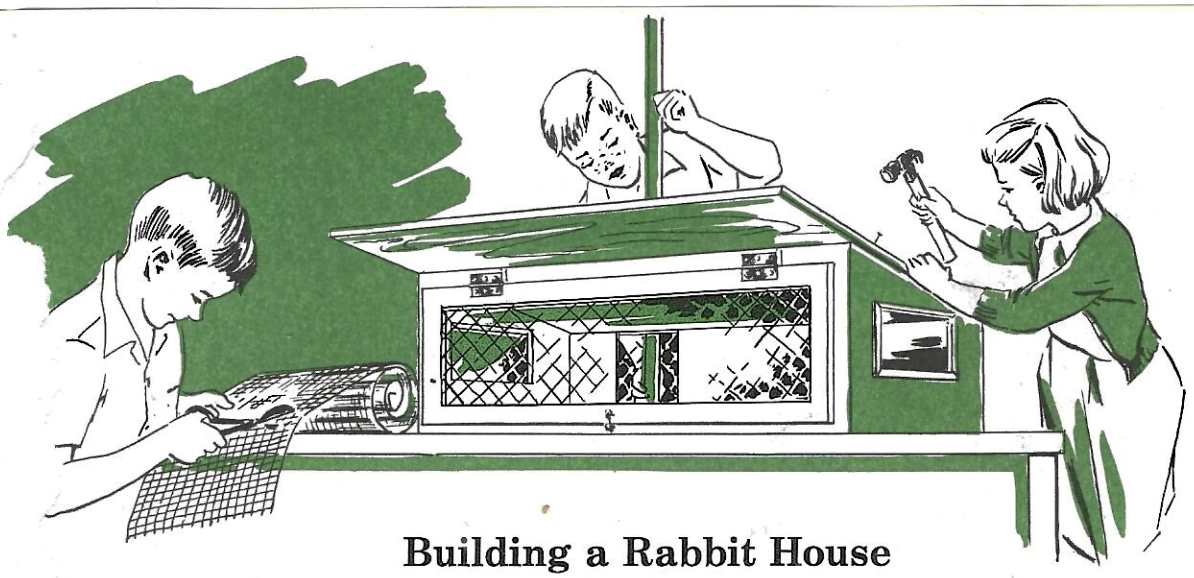
$\frac{1}{4} = \frac{?}{24}$ $\qquad$ $\frac{2}{3} = \frac{?}{24}$

**4.** Change to lowest common denominator:



$\frac{2}{3}, \frac{5}{6}, \frac{7}{12} = ?$

$\frac{1}{2}, \frac{3}{4}, \frac{5}{8} = ?$



## Building a Rabbit House

Jimmy, Paul, and Kathy are building a house for the sixth grade's pet rabbit.

If they nail 2 pieces of plywood, each $\frac{1}{4}$ of a yard wide, side by side, how wide a space will the boards cover?

$\boxed{\frac{1}{4}\text{ yd.}}$ $\boxed{\frac{1}{4}\text{ yd.}}$ Add $\frac{1}{4}$ and $\frac{1}{4}$: $\frac{1}{4} + \frac{1}{4} = ?$

$$\begin{array}{r} \frac{1}{4} \\ +\ \frac{1}{4} \\ \hline \frac{2}{4} = \frac{1}{2} \end{array}$$

The size of the unit of measure is the fourth. Why? When you add, you find the sum of the how-many of these units. (Why would you not add the 4's?)

So, in this problem you have $1 + 1 = 2$, or 2 units of the $\frac{1}{4}$ size. The answer then is $\frac{2}{4}$ yd. This value can be changed to $\frac{1}{2}$ yd. How? When, in adding fractions, you have the L.C.D., it is good practice to write this denominator of the sum first. Then draw a line over it, writing the sum of the numerators above this line.

Add:

| | a. | b. | c. | d. | e. | f. | g. | h. |
|---|---|---|---|---|---|---|---|---|
| **1.** | $\frac{1}{5}$ | $\frac{1}{8}$ | $\frac{2}{7}$ | $\frac{3}{9}$ | $\frac{1}{4}$ | $\frac{1}{3}$ | $\frac{2}{6}$ | $\frac{1}{9}$ |
| | $\frac{1}{5}$ | $\frac{4}{8}$ | $\frac{4}{7}$ | $\frac{5}{9}$ | $\frac{2}{4}$ | $\frac{2}{3}$ | $\frac{3}{6}$ | $\frac{4}{9}$ |
| **2.** | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{1}{6}$ | $\frac{1}{5}$ | $\frac{2}{9}$ | $\frac{2}{5}$ | $\frac{3}{7}$ | $\frac{3}{8}$ |
| | $\frac{2}{4}$ | $\frac{1}{2}$ | $\frac{2}{6}$ | $\frac{3}{5}$ | $\frac{6}{9}$ | $\frac{2}{5}$ | $\frac{2}{7}$ | $\frac{4}{8}$ |

## Adding Unlike Fractions—L.C.D.

Two boards, $\frac{1}{2}$ yd. wide and $\frac{1}{8}$ yd. wide, were placed side by side. How could the children building the rabbit house add these widths?

$$\begin{array}{cc} \frac{1}{2} & \frac{4}{8} \\ & = \\ +\frac{1}{8} & +\frac{1}{8} \\ \hline \frac{5}{8} & \frac{5}{8} \end{array}$$

You cannot add these fractions as they are because they have different parts or units. (They are unlike fractions.) When you change them to like fractions using the L.C.D., you can add. The answer is $\frac{5}{8}$ yd.

Do you see that these unlike fractions can be added if you change the unit in one of them to the unit in the other? When this is possible, it is called adding fractions with the L.C.D. present.

**1.** What would be the L.C.D. in these pairs of fractions?

$\frac{1}{2}$ and $\frac{1}{6}$ $\quad$ $\frac{3}{4}$ and $\frac{5}{12}$ $\quad$ $\frac{2}{3}$ and $\frac{5}{9}$ $\quad$ $\frac{3}{4}$ and $\frac{5}{16}$

**2.** How much money is $\frac{1}{10}$ of a dollar? How much money is $\frac{3}{10}$ of a dollar? $\frac{2}{5}$ of a dollar?

**3.** Add $\frac{3}{10}$ of a dollar and $\frac{2}{5}$ of a dollar. Change the sum of the fractions to a value in money.

Use this method:

$$\begin{array}{cc} \frac{3}{10} & \frac{?}{10} \\ & = \\ +\frac{2}{5} & +\frac{?}{10} \\ \hline & \frac{?}{10} \end{array}$$

How do you change $\frac{2}{5}$ to tenths?

Why is 10 the L.C.D. for the two fractions?

$\frac{?}{10}$ of a dollar = ? cents.

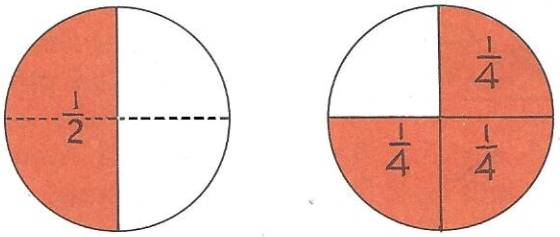
**4.** Add (changing any sums possible to lowest terms):

$\frac{1}{3} + \frac{4}{9}$ $\quad$ $\frac{1}{5} + \frac{3}{10}$ $\quad$ $\frac{1}{4} + \frac{7}{12}$ $\quad$ $\frac{1}{6} + \frac{1}{3}$ $\quad$ $\frac{1}{2} + \frac{3}{8}$

**5.** Add $\frac{1}{2}$ and $\frac{3}{4}$:

You know that you must have like fractions if you want to do an addition. To add $\frac{1}{2}$ and $\frac{3}{4}$, you first must find the L.C.D.

$$\begin{array}{r} \frac{1}{2} \\ +\frac{3}{4} \\ \hline \end{array}$$

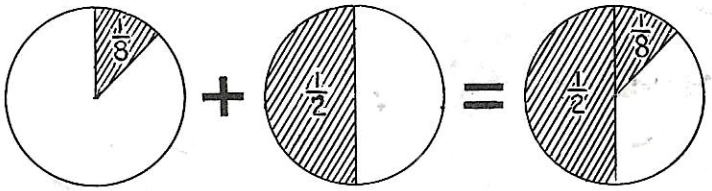



To add $\frac{1}{2}+\frac{3}{4}$, find the L.C.D. which is 4, and you get $\frac{2}{4}+\frac{3}{4}=?$ in the following manner:

$$\begin{array}{cc} \frac{1}{2} & \frac{2}{4} \\ & = \\ +\frac{3}{4} & +\frac{3}{4} \\ & \frac{5}{4}=\frac{4}{4}+\frac{1}{4}=1+\frac{1}{4}=1\frac{1}{4} \end{array}$$

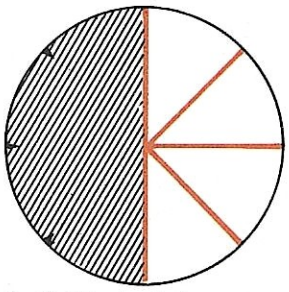
$\frac{5}{4}$ is an improper fraction. It is changed to $1\frac{1}{4}$, a mixed number. How is this done?

**6.** Add $\frac{1}{8}$ and $\frac{1}{2}$:

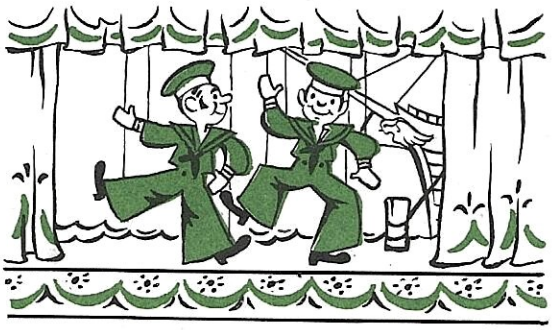



What is the L.C.D.? Can $\frac{1}{2}$ be measured in $\frac{1}{8}$'s? How? Look at this picture:

$$\begin{array}{cc} \frac{1}{8} & \frac{1}{8} \\ & = \\ +\frac{1}{2} & +\frac{4}{8} \\ & \frac{5}{8} \end{array}$$



Is this statement true? The L.C.D. is 8 because it can be divided evenly by both 2 and 8. Why?

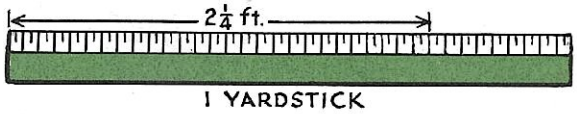


## The Puppet Show

The sixth-grade class in the Brown School finished making some puppets. They decided to give a show for the entire school.

The boys and girls first made a stage for the show. They wanted 4 curtains for the stage and found that each curtain should be 2 feet and 3 inches long.

Harry said, "2 feet and 3 inches could be called 2 ft. and $\frac{1}{4}$ of another foot. I could write it as $2\frac{1}{4}$ ft. That's a mixed number."




Joe said, "That length on this yardstick equals $\frac{3}{4}$ yd."

Were both boys right in what they said? 3 in. = $\frac{1}{4}$ ft. Why? 2 ft. + 3 in. = 2 ft. + $\frac{1}{4}$ ft. = $2\frac{1}{4}$ ft. Why?

2 ft. + 3 in. = 27 in. Why?

27 in. = $\frac{27}{36}$ yd. = $\frac{3}{4}$ yd. Why?

**1.** Harry said that he had a mixed number. How are mixed numbers found in measurements? Tell how you use equal parts of a unit to measure a length less than a whole unit.

**2.** Could $2\frac{1}{4}$ ft. be measured entirely in units of $\frac{1}{4}$ ft.? How many $\frac{1}{4}$'s are there in 1 whole? in 2 wholes? in $2\frac{1}{4}$ units?

**3.** Why do we call $\frac{9}{4}$ an improper fraction?

**4.** Write the number 2 as an improper fraction.

**5.** If the children had $2\frac{1}{3}$ yd. of curtain material, what improper fraction could they write for this value?

**6.** Change the numbers 3 and $\frac{3}{4}$ to fractions with an L.C.D.

**7.** How many eighths are there in 2? in $2\frac{5}{8}$? in $2\frac{3}{4}$?

**8.** $1\frac{2}{3} = \frac{?}{3}$ $\quad 3\frac{3}{4} = \frac{?}{4}$ $\quad 2\frac{1}{5} = \frac{?}{5}$ $\quad 3\frac{1}{2} = \frac{?}{2}$ $\quad 1\frac{5}{6} = \frac{?}{6}$

## Fractional Parts Changed to Wholes and Parts

Suppose Sue needed 14 ft. of goods which are sold by the yard. Can 14 ft. be written as the improper fraction $\frac{14}{3}$ yd.?

Sue should change this improper fraction into a mixed number before she gives her order to the store clerk. Why?

Remember: An improper fraction is a value that is equal to or greater than one whole unit. (There are at least as many parts in use as there are in 1 whole unit.)

When both the numerator and denominator of a fraction are the same number, there are as many of the parts in use as there are parts in the whole. This improper fraction then is equal to a whole, or 1. Thus $\frac{5}{5} = 1$.

Why is $\frac{7}{7}$ called an improper fraction? Does $\frac{7}{7} = 1$?

It takes 3 thirds to make a whole. There will be as many wholes in $\frac{14}{3}$ as $3\overline{)14}$.

The ones left over make a fraction: $\frac{2}{3}$ yd. left over. Why?

Thus, *to change an improper fraction to a mixed number, you can divide the numerator by the denominator.*

Change the following improper fractions to mixed numbers:

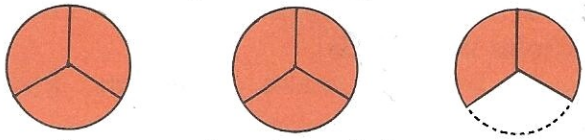
| | a. | b. | c. | d. |
|---|---|---|---|---|
| **1.** | $\frac{5}{3} = ?$ | $\frac{6}{4} = ?$ | $\frac{24}{7} = ?$ | $\frac{6}{3} = ?$ |
| **2.** | $\frac{32}{5} = ?$ | $\frac{13}{4} = ?$ | $\frac{14}{4} = ?$ | $\frac{8}{4} = ?$ |
| **3.** | $\frac{13}{7} = ?$ | $\frac{19}{3} = ?$ | $\frac{17}{9} = ?$ | $\frac{5}{5} = ?$ |
| **4.** | $\frac{28}{5} = ?$ | $\frac{26}{8} = ?$ | $\frac{21}{5} = ?$ | $\frac{12}{9} = ?$ |
| **5.** | $\frac{33}{4} = ?$ | $\frac{11}{4} = ?$ | $\frac{15}{2} = ?$ | $\frac{21}{5} = ?$ |
| **6.** | $\frac{12}{8} = ?$ | $\frac{32}{12} = ?$ | $\frac{11}{3} = ?$ | $\frac{17}{8} = ?$ |

## Comparing Improper Fractions and Mixed Numbers

Linda fed her dog $\frac{1}{3}$ of a pound of meat each day. Nancy agreed to keep Linda's dog and feed him while she was on vacation. Linda was to provide the meat. How many lb. of meat would she have to buy to feed her dog for 8 days?

Is $\frac{8}{3}$ an improper fraction? Why? If you change $\frac{8}{3}$ to a mixed number, it will give you the number of pounds that Linda will have to buy. The fraction $\frac{8}{3}$ means that there are 8 parts the size of thirds. Three parts will make a whole: $\frac{3}{3} = 1$. $\frac{8}{3} = ?$. If it takes 3 of the thirds to make a whole, then you must divide 8 by 3 to find the number of groups of 3 parts that there are in 8. There are two groups of three with 2 left over which is $\frac{2}{3}$ of a whole.

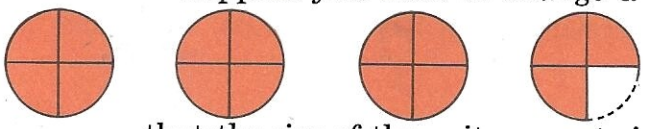
$$3\overline{)8} \quad 2\tfrac{2}{3} \quad 6 \quad 2$$



$\frac{8}{3} = 2\frac{2}{3}$

$\frac{8}{3}$ is an improper fraction. It becomes $2\frac{2}{3}$ when changed to a mixed number.

Suppose you want to change a mixed number into an improper fraction.



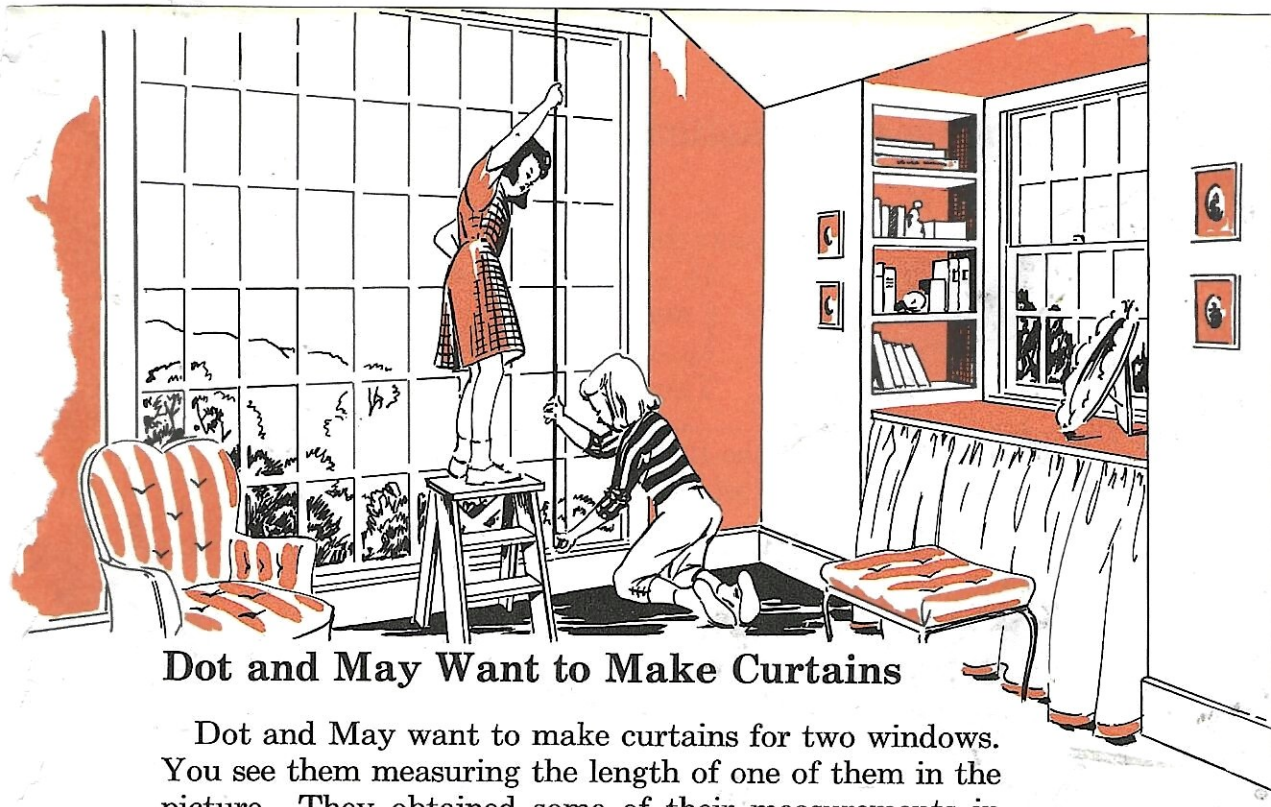
$3\frac{3}{4}$ is a mixed number. Why? You know that the size of the units or parts is $\frac{1}{4}$. How do you know? $\frac{4}{4} = 1$; therefore you know that there are $4 \times 3$ or 12 parts in 3. Why? You then have $\frac{12}{4} + \frac{3}{4}$. Why? $3\frac{3}{4} = \frac{15}{4}$.

Change to mixed numbers:

| **1.** | **a.** | **b.** | **c.** | **d.** | **e.** | **f.** | **g.** | **h.** |
|---|---|---|---|---|---|---|---|---|
| | $\frac{9}{2}$ | $\frac{12}{5}$ | $\frac{11}{4}$ | $\frac{17}{8}$ | $\frac{14}{3}$ | $\frac{29}{12}$ | $\frac{23}{6}$ | $\frac{37}{10}$ |

Change to improper fractions:

| **2.** | **a.** | **b.** | **c.** | **d.** | **e.** | **f.** | **g.** | **h.** |
|---|---|---|---|---|---|---|---|---|
| | $2\frac{1}{4}$ | $3\frac{3}{8}$ | $5\frac{2}{3}$ | $1\frac{7}{8}$ | $4\frac{5}{6}$ | $3\frac{1}{12}$ | $3\frac{3}{4}$ | $2\frac{7}{9}$ |



## Dot and May Want to Make Curtains

Dot and May want to make curtains for two windows. You see them measuring the length of one of them in the picture. They obtained some of their measurements in yards, some in feet, and some in inches.

First, they changed all their measurements to yards. The girls wanted the same material for both windows. They found that one window was $1\frac{1}{2}$ yd. long and the other was $2\frac{1}{4}$ yd. long. How much material must they buy altogether?

$$\begin{array}{r} 1\frac{1}{2} \text{ yd.} \\ +\ 2\frac{1}{4} \text{ yd.} \\ \hline ? \end{array}$$

You cannot add the fractional parts as they are given. Why?

Are $\frac{1}{2}$ and $\frac{1}{4}$ unlike fractions? How do you know?

If you change the size of the parts so that they become like fractions, you can add them. Why?

$$\begin{array}{r} 1\frac{1}{2} \text{ yd.} \\ +\ 2\frac{1}{4} \text{ yd.} \end{array} \quad = \quad \begin{array}{r} 1\frac{2}{4} \text{ yd.} \\ +\ 2\frac{1}{4} \text{ yd.} \\ \hline 3\frac{3}{4} \text{ yd.} \end{array}$$

You can change halves to fourths. (You usually have to change to the smaller part when adding unlike fractions.) Why is this change of $\frac{1}{2}$ to $\frac{2}{4}$ called raising to higher terms? Add the fractional parts first.

## Changing the Fractional Units in Answers

**1.** $1\frac{1}{4} + 2\frac{3}{4} = ?$

Add the fractional parts first:

$$\begin{array}{r} 1\frac{1}{4} \\ +\ 2\frac{3}{4} \\ \hline 3\frac{4}{4} = 4 \end{array}$$

$1\frac{1}{4} + \frac{3}{4} = \frac{4}{4}$. You see that the $\frac{4}{4}$ makes 1 whole.

$1\frac{1}{4} + 2\frac{3}{4} = 3 + \frac{4}{4} = ?$

Thus your sum is 4.

> When the sum of the fractional parts is a whole or a mixed number, the whole part is added to the sum of the other wholes.

**2.** $1\frac{1}{8} + 2\frac{3}{4} + 3\frac{3}{8} = ?$

$$\begin{array}{r} 1\frac{1}{8} \\ 2\frac{3}{4} \\ +\ 3\frac{3}{8} \\ \hline 7\frac{1}{4} \end{array}$$

First, add the fractional parts:

$\frac{1}{8} + \frac{6}{8} + \frac{3}{8} = \frac{10}{8} = \frac{5}{4}$ or $1\frac{1}{4}$.

Why does $\frac{5}{4} = 1\frac{1}{4}$?

When you change $\frac{5}{4}$ to $1\frac{1}{4}$, you have put your answers in a form that is used most commonly. Write the fractional part of this in the place for fractions, and hold the 1. Adding the wholes, $1 + 2 + 3 + 1$ (held) $= 7$. The sum is $7\frac{1}{4}$.

**1.** Billy wants to make two shelves for his room. How long a board must he buy if one shelf is to be 2 ft. $6\frac{1}{2}$ in. long and the other shelf is to be 4 ft. $3\frac{3}{4}$ in. long?

**2.** Mary is making a skirt and a blouse to match. How much material must she order if she needs 3 yd. $4\frac{1}{2}$ in. for the skirt, and $18\frac{5}{8}$ in. for the blouse?

Find the sum:

| | a. | b. | c. | d. | e. | f. | g. |
|---|---|---|---|---|---|---|---|
| **3.** | $2\frac{5}{6}$ | $\frac{4}{5}$ | $5\frac{7}{10}$ | $6\frac{5}{6}$ | $3\frac{1}{4}$ | $8\frac{1}{4}$ | $6\frac{1}{8}$ |
| | $3\frac{7}{12}$ | $3\frac{7}{10}$ | $\frac{1}{2}$ | $2\frac{1}{3}$ | $2\frac{7}{12}$ | $2\frac{3}{4}$ | $1\frac{7}{8}$ |
| **4.** | $1\frac{2}{3}$ | $\frac{3}{5}$ | $2\frac{1}{3}$ | $7\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{1}{2}$ | $3\frac{3}{4}$ |
| | $5\frac{1}{2}$ | $1\frac{1}{2}$ | $1\frac{1}{2}$ | $5$ | $2\frac{1}{2}$ | $\frac{1}{3}$ | $1\frac{2}{3}$ |
| | $2\frac{7}{12}$ | $3\frac{3}{10}$ | $3\frac{1}{6}$ | $3\frac{3}{4}$ | $1\frac{3}{4}$ | $2\frac{1}{6}$ | $\frac{7}{12}$ |

## Practice Exercises

**1.** Add the following, giving your answers in simplest terms:

| **a.** | **b.** | **c.** | **d.** | **e.** | **f.** | **g.** |
|---|---|---|---|---|---|---|
| $1\frac{2}{3}$ | $1\frac{3}{4}$ | $1\frac{5}{8}$ | $1\frac{5}{9}$ | $1\frac{5}{16}$ | $\frac{7}{8}$ | $2\frac{7}{16}$ |
| $2\frac{5}{6}$ | $3\frac{3}{8}$ | $3\frac{1}{2}$ | $3\frac{2}{3}$ | $2\frac{11}{16}$ | $1\frac{1}{2}$ | $\frac{3}{4}$ |

**2.** Tell how measuring might give these values and why you would want to change them:

**a.** the parts of a foot: $\frac{1}{2} + \frac{1}{4} = \frac{6}{12} + \frac{3}{12} = \frac{?}{12} =$ ? in.

**b.** the parts of a pound: $\frac{1}{4} + \frac{1}{8} = \frac{4}{16} + \frac{2}{16} = \frac{6}{16} =$ ? oz.

**c.** the parts of a yard: $\frac{2}{3} + \frac{1}{12} = \frac{24}{36} + \frac{?}{36} =$ ? in.

Add:

| | **a.** | **b.** | **c.** | **d.** | **e.** | **f.** | **g.** |
|---|---|---|---|---|---|---|---|
| **3.** | $2\frac{5}{6}$ | $4\frac{3}{5}$ | $4\frac{5}{12}$ | $3\frac{1}{4}$ | $2\frac{9}{10}$ | $3\frac{1}{2}$ | $7\frac{2}{3}$ |
| | $1\frac{1}{3}$ | $2\frac{2}{5}$ | $3\frac{1}{4}$ | $2\frac{3}{8}$ | $1\frac{3}{5}$ | $4\frac{3}{4}$ | $5\frac{1}{6}$ |
| **4.** | $1\frac{2}{3}$ | $3\frac{3}{4}$ | $7\frac{1}{5}$ | $5\frac{2}{9}$ | $1\frac{4}{5}$ | $\frac{5}{9}$ | $7\frac{5}{8}$ |
| | $4\frac{5}{6}$ | $5\frac{1}{2}$ | $2\frac{7}{10}$ | $6\frac{2}{3}$ | $2\frac{7}{10}$ | $2\frac{2}{3}$ | $2\frac{3}{4}$ |
| **5.** | $3\frac{1}{4}$ | $6\frac{3}{8}$ | $4\frac{1}{3}$ | $6\frac{3}{8}$ | $3\frac{1}{2}$ | $3\frac{1}{4}$ | $6\frac{2}{5}$ |
| | $5\frac{1}{2}$ | $2\frac{1}{4}$ | $2\frac{5}{12}$ | $7\frac{1}{16}$ | $4\frac{9}{10}$ | $1\frac{5}{16}$ | $5\frac{3}{5}$ |

**6. a.** $3\frac{5}{7} + 2\frac{2}{7} =$ ? **b.** $4\frac{5}{9} + 5\frac{11}{18} =$ ? **c.** $1\frac{5}{7} + 3\frac{9}{14} =$ ?

**7. a.** $\frac{9}{8} + 1\frac{3}{8} =$ ? **b.** $\frac{11}{6} + \frac{2}{3} =$ ? **c.** $4 + 1\frac{2}{5} + 3\frac{9}{10} =$ ?

Add:

| **8.** | **a.** | **b.** | **c.** | **d.** | **e.** | **f.** | **g.** | **h.** |
|---|---|---|---|---|---|---|---|---|
| | $\frac{3}{4}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $1\frac{1}{2}$ | $\frac{2}{3}$ | $1\frac{1}{6}$ | 5 | $3\frac{1}{2}$ |
| | $\frac{1}{2}$ | $\frac{5}{6}$ | $\frac{3}{16}$ | $\frac{7}{10}$ | $\frac{11}{12}$ | $1\frac{1}{2}$ | $3\frac{5}{12}$ | 5 |
| | $\frac{5}{12}$ | $\frac{5}{12}$ | $1\frac{1}{2}$ | $\frac{3}{5}$ | $1\frac{1}{4}$ | $1\frac{1}{3}$ | $2\frac{3}{4}$ | $1\frac{7}{10}$ |

Do these additions in your head (give answers orally). Then check your answers with written work.

| | **a.** | **b.** | **c.** | **d.** | **e.** |
|---|---|---|---|---|---|
| **9.** | $\frac{1}{2} + \frac{9}{10}$ | $\frac{3}{8} + 1\frac{1}{2}$ | $2\frac{2}{3} + 1\frac{5}{6}$ | $4\frac{9}{16} + 1\frac{1}{2}$ | $2\frac{7}{8} + \frac{5}{6}$ |
| **10.** | $\frac{4}{5} + \frac{3}{10}$ | $\frac{2}{3} + \frac{5}{12}$ | $1\frac{1}{4} + 1\frac{1}{8}$ | $1\frac{3}{4} + 1\frac{11}{16}$ | $2\frac{4}{5} + 1\frac{3}{10}$ |

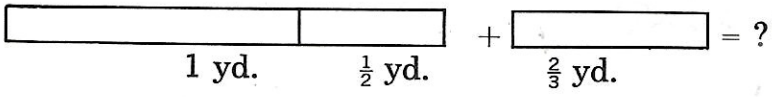


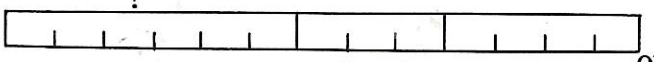
## Jane's Quilting Party

Jane and some of her friends are making quilts. They have different sizes of cloth which must be fitted together. They find it necessary to measure in wholes and in parts of units, and to add fractions so that the patterns in these quilts will come out even. For the cloth to which the pattern materials will be sewed, they found that they must sew a $1\frac{1}{2}$-yd. piece and $\frac{2}{3}$-yd. piece together. How long will the cloth be after the two pieces are joined?

Let's look at the addition by using diagrams:

$1\frac{1}{2}$
$+\ \frac{2}{3}$
$?$




or $1\frac{3}{6} = 2\frac{1}{6}$ Why?
$+\ \frac{4}{6}$

**1.** The lowest common denominator for halves and thirds is sixths. Why?

**2.** Tell what the girls would have to do in order to find the total length of two pieces if one is $1\frac{1}{3}$ yd. and the other is $1\frac{1}{4}$ yd. long.

$1\frac{1}{3} = 1\frac{4}{12}$
$+\ 1\frac{1}{4} \quad +\ 1\frac{3}{12}$
$? \quad 1\frac{7}{12}$

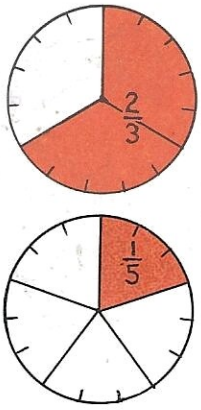
In adding these fractions you must change both of them to a common unit.

$\frac{1}{3} = \frac{4}{12}$ $\qquad$ $\frac{1}{4} = \frac{3}{12}$

How did you find the L.C.D.?

*Adding Mixed Numbers, L.C.D. Not Present*

## Generalizing the L.C.D.




If you want to add $\frac{2}{3}$ and $\frac{1}{5}$, you must find the L.C.D. In measurements, this means to find the largest unit in which both of the given units, thirds and fifths, can be measured. In numbers, it means to find the smallest number that can be divided by both 3 and 5 evenly. The L.C.D. is 15. It is found by multiplying the largest denominator by 2, 3, or some other number to find the smallest number into which all of the denominators can be divided evenly.

Thirds can be changed to smaller parts of the size fifteenths. How?

Fifths can be changed to smaller parts of the size fifteenths. How?

How many fifteenths in $\frac{1}{3}$? in $\frac{2}{3}$? in $\frac{1}{5}$?

In parts like these, where you have to change both of them, do you see that their product gives the L.C.D.?

$$\begin{array}{r} \frac{2}{3} \\ +\frac{1}{5} \end{array} = \begin{array}{r} \frac{10}{15} \\ +\frac{3}{15} \end{array} = \frac{13}{15}$$

Count these parts in the drawing above to check the answer.

In fractions like $\frac{3}{8}$ and $\frac{5}{6}$, the L.C.D. is not their product. Can you tell why? ($2 \times 8 = 16$, $3 \times 8 = 24$) Is 24 the smallest number that can be divided evenly by both 8 and 6?

Changing the denominator of a fraction to an L.C.D. is usually the process of changing it to higher terms. Why?

What must you multiply the denominator of $\frac{2}{3}$ by to change it to an L.C.D. of 9? What must you multiply the numerator by in order to keep the same value of the fraction?

$$\begin{array}{r} \frac{3}{5} \\ +\frac{2}{3} \end{array} = \begin{array}{r} \frac{9}{15} \\ +\frac{10}{15} \end{array}$$

Of course, you can think this way: The L.C.D. is 15. Then, $15 \div 5 = 3$, and $3 \times 3 = 9$. 9 is the new numerator. Try this for the fraction $\frac{2}{3}$. This is the general method. State this general method in words.

## Practice in Adding Mixed Numbers

Mixed numbers can be added by first changing them to improper fractions. Study this example:

$$\begin{array}{r} 2\frac{2}{3} \\ +\,4\frac{1}{6} \end{array} = \begin{array}{r} \frac{8}{3} \\ +\,\frac{25}{6} \end{array} = \begin{array}{r} \frac{16}{6} \\ +\,\frac{25}{6} \end{array} = \frac{41}{6} = 6\frac{5}{6}$$

Notice that the answer has been changed back to a mixed number.

Add as mixed numbers:

| | a. | b. | c. | d. | e. | f. | g. |
|---|---|---|---|---|---|---|---|
| 1. | $1\frac{2}{3}$ | $3\frac{3}{8}$ | $2\frac{2}{5}$ | $3\frac{1}{6}$ | $7\frac{2}{3}$ | $4\frac{1}{8}$ | $1\frac{7}{8}$ |
| | $2\frac{5}{6}$ | $5\frac{1}{4}$ | $4\frac{7}{10}$ | $1\frac{5}{12}$ | $4\frac{5}{9}$ | $3\frac{9}{16}$ | $2\frac{3}{4}$ |
| 2. | $4\frac{3}{4}$ | $1\frac{1}{6}$ | $5\frac{1}{3}$ | $2\frac{1}{4}$ | $3\frac{1}{6}$ | $1\frac{5}{8}$ | $1\frac{3}{5}$ |
| | $2\frac{2}{3}$ | $3\frac{3}{4}$ | $6\frac{1}{5}$ | $4\frac{3}{5}$ | $5\frac{1}{9}$ | $3\frac{7}{12}$ | $2\frac{1}{4}$ |

Add as improper fractions:

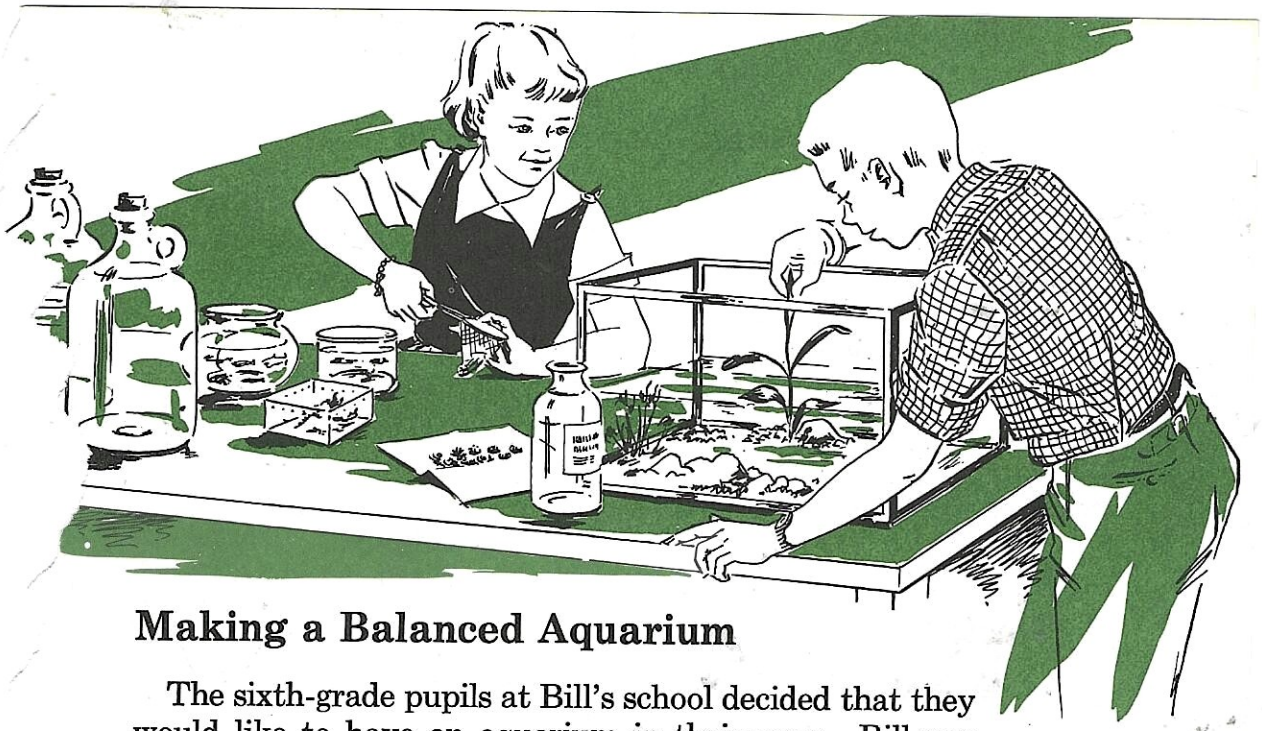
| | a. | b. | c. | d. | e. | f. | g. |
|---|---|---|---|---|---|---|---|
| 3. | $1\frac{3}{4}$ | $4\frac{5}{6}$ | $3\frac{1}{2}$ | $2\frac{2}{3}$ | $6\frac{5}{6}$ | $5\frac{3}{8}$ | $2\frac{5}{9}$ |
| | $4\frac{5}{8}$ | $2\frac{1}{3}$ | $5\frac{3}{4}$ | $1\frac{1}{9}$ | $4\frac{1}{12}$ | $3\frac{1}{4}$ | $3\frac{1}{3}$ |
| 4. | $2\frac{1}{2}$ | $4\frac{5}{6}$ | $5\frac{1}{4}$ | $3\frac{5}{6}$ | $6\frac{2}{3}$ | $1\frac{5}{12}$ | $2\frac{1}{6}$ |
| | $3\frac{1}{3}$ | $1\frac{3}{4}$ | $2\frac{2}{3}$ | $2\frac{1}{9}$ | $5\frac{3}{5}$ | $4\frac{3}{8}$ | $3\frac{7}{10}$ |

5. $2\frac{2}{7} + 3\frac{2}{3} = ?$ $\quad 4\frac{2}{5} + 3\frac{3}{4} = ?$ $\quad 4\frac{1}{2} + 3\frac{2}{9} = ?$

Find the sum:

| | a. | b. | c. | d. |
|---|---|---|---|---|
| 6. | $\frac{2}{3} + \frac{3}{4}$ | $1\frac{2}{3} + 2\frac{3}{4}$ | $2\frac{3}{8} + 5\frac{11}{16}$ | $3\frac{4}{5} + 5\frac{9}{10}$ |

| | a. | b. | c. | d. | e. | f. | g. |
|---|---|---|---|---|---|---|---|
| 7. | $2\frac{1}{2}$ | $2\frac{4}{5}$ | $\frac{1}{3}$ | $3\frac{3}{8}$ | $\frac{1}{2}$ | $5\frac{5}{6}$ | $2\frac{1}{2}$ |
| | $1\frac{2}{3}$ | $\frac{1}{2}$ | $2\frac{3}{4}$ | $5\frac{1}{2}$ | $2\frac{3}{8}$ | $\frac{1}{2}$ | $1\frac{3}{4}$ |
| | $3\frac{5}{12}$ | $3\frac{3}{10}$ | $2\frac{5}{6}$ | $\frac{5}{16}$ | $\frac{3}{4}$ | $\frac{2}{3}$ | $\frac{5}{6}$ |
| 8. | $8\frac{1}{3}$ | $2\frac{1}{7}$ | $5\frac{2}{5}$ | $3\frac{1}{3}$ | $15\frac{2}{9}$ | $5\frac{3}{8}$ | $\frac{4}{9}$ |
| | $\frac{5}{6}$ | $4\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{3}{5}$ | $6\frac{1}{2}$ | $7\frac{5}{8}$ | $7\frac{5}{9}$ |



## Making a Balanced Aquarium

The sixth-grade pupils at Bill's school decided that they would like to have an aquarium in their room. Bill was on the committee to find out what they needed.

Jane said, "The aquarium must be balanced. I read that in a balanced aquarium, $\frac{1}{3}$ of the number of animals should be fish, and $\frac{2}{3}$ should be baby snails and salamanders. The book said that no more than $\frac{1}{6}$ should be salamanders."

The pupils then wanted to know what part should be snails. They found that they would have to subtract. Why?

$$\begin{array}{r} \frac{2}{3} \\ -\frac{1}{6} \\ \hline ? \end{array}$$

Subtraction of fractions is like addition in the following ways:

**a.** You must have like fractions before you subtract.

**b.** When like fractions are obtained, the common denominator should be the L.C.D.

$$\begin{array}{rr} \frac{2}{3} = & \frac{4}{6} \\ -\frac{1}{6} & -\frac{1}{6} \\ \hline & ? \end{array}$$

Why is the L.C.D. the number 6 here? How do you get $\frac{4}{6}$ from the fraction $\frac{2}{3}$? $\frac{4}{6} - \frac{1}{6} = \frac{?}{6}$. Bill said $\frac{1}{2}$ the animals should be snails. Was he right?

## Problems and Exercises

**1.** Ann saved $\frac{1}{3}$ of her allowance and $\frac{1}{5}$ of what she earned at the bakery each Saturday for her clothing budget. If her allowance and her earnings per week were equal amounts, which item gave her more money for her budget? How much more is $\frac{1}{3}$ than $\frac{1}{5}$?

$$\frac{1}{3} = \frac{?}{15} \qquad \frac{1}{5} = \frac{?}{15} \qquad \frac{1}{3} - \frac{1}{5} = \frac{?}{15} - \frac{?}{15} = ?$$

**2.** Joan figured that she spent $\frac{1}{4}$ of her time doing problems, and $\frac{2}{5}$ of her time reading. How much more of her time was spent reading than was spent doing problems?

**3.** If a large candy bar weighs $\frac{1}{4}$ pound and a small one weighs $\frac{1}{8}$ of a pound, how much more does the larger one weigh?

**4.** Bob's house is $\frac{1}{5}$ of a mile from the post office while Art's house is $\frac{9}{10}$ of a mile. How far does Art live from Bob if they live on the same street in the same direction from the post office?

**5.** In our room $\frac{3}{8}$ of the children are girls. How many more boys are there than girls, expressed as a fractional part of the class enrollment?

**6.** Jane bought $\frac{3}{4}$ of a yard of taffeta and $\frac{5}{6}$ of a yard of rayon. How much more rayon than taffeta did she buy?

| | | | | | |
|---|---|---|---|---|---|
| **7.** $\frac{5}{7}$ | **8.** $\frac{6}{12}$ | **9.** $\frac{5}{8}$ | **10.** $\frac{4}{9}$ | **11.** $\frac{6}{8}$ | **12.** $\frac{8}{12}$ |
| $-\frac{2}{7}$ | $-\frac{3}{12}$ | $-\frac{2}{8}$ | $-\frac{3}{9}$ | $-\frac{1}{2}$ | $-\frac{2}{3}$ |
| **13.** $\frac{3}{4}$ | **14.** $\frac{15}{32}$ | **15.** $\frac{3}{4}$ | **16.** $\frac{7}{8}$ | **17.** $\frac{4}{5}$ | **18.** $\frac{4}{5}$ |
| $-\frac{1}{4}$ | $-\frac{5}{16}$ | $-\frac{1}{2}$ | $-\frac{3}{4}$ | $-\frac{3}{10}$ | $-\frac{2}{3}$ |

## Practice for Skill—Adding Fractions

You remember that to add like fractions, you add the numerators: $\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$

2 sevenths
+ 3 sevenths
5 sevenths

You add three like fractions in the same way: $\frac{1}{9} + \frac{3}{9} + \frac{4}{9} = \frac{?}{9}$

1 ninth
3 ninths
+ 4 ninths
? ninths

Add these fractions:

**1.** $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$

**2.** $\frac{1}{11}$, $\frac{3}{11}$, $\frac{5}{11}$

**3.** $\frac{1}{4}$, $\frac{1}{4}$, $\frac{3}{4}$

**4.** $\frac{1}{15}$, $\frac{5}{15}$, $\frac{7}{15}$

**5.** $\frac{4}{11}$, $\frac{3}{11}$, $\frac{2}{11}$

**6.** $\frac{3}{13}$, $\frac{4}{13}$, $\frac{5}{13}$

Add the following:

**7.** $\frac{2}{5} + \frac{1}{3} = ?$ $\quad \frac{2}{5} = \frac{6}{15}$, $+\frac{1}{3} = +\frac{5}{15}$

**8.** $\frac{1}{6} + \frac{1}{4} + \frac{1}{3} = ?$ $\quad \frac{1}{6} = \frac{?}{12}$, $\frac{1}{4} = \frac{?}{12}$, $+\frac{1}{3} = +\frac{?}{12}$

| | a. | b. | c. | d. | e. | f. |
|---|---|---|---|---|---|---|
| **9.** | $\frac{2}{3} + \frac{3}{7}$ | $\frac{2}{3} + \frac{4}{5}$ | $\frac{4}{11} + \frac{6}{11}$ | $\frac{3}{4} + \frac{5}{12}$ | $\frac{1}{8} + \frac{1}{4}$ | $\frac{2}{5} + \frac{3}{10}$ |
| **10.** | $\frac{3}{9}$, $\frac{2}{9}$, $+\frac{4}{9}$ | $\frac{5}{11}$, $\frac{3}{11}$, $+\frac{2}{11}$ | $\frac{3}{10}$, $\frac{9}{20}$, $+\frac{5}{20}$ | $\frac{3}{12}$, $\frac{5}{24}$, $+\frac{7}{24}$ | $\frac{3}{8}$, $\frac{1}{4}$, $+\frac{3}{16}$ | $\frac{1}{6}$, $\frac{5}{12}$, $+\frac{5}{24}$ |
| **11.** | $\frac{1}{4}$, $\frac{1}{3}$, $+\frac{1}{2}$ | $\frac{2}{3}$, $\frac{3}{5}$, $+\frac{1}{3}$ | $\frac{3}{8}$, $\frac{5}{16}$, $+\frac{1}{4}$ | $\frac{1}{6}$, $\frac{3}{12}$, $+\frac{1}{4}$ | $\frac{5}{6}$, $\frac{2}{9}$, $+\frac{1}{3}$ | $\frac{2}{5}$, $\frac{3}{4}$, $+\frac{7}{10}$ |

**12.** $\frac{2}{7} + \frac{2}{3} = \frac{?}{21} + \frac{?}{21} = ?$

**13.** $\frac{3}{8} + \frac{2}{3} + \frac{5}{6} = \frac{?}{24} + \frac{?}{24} + \frac{?}{24} = ?$

**14.** In $\frac{5}{6} + \frac{3}{5}$, what is wrong with the following?

$$\frac{5}{6} + \frac{3}{5} = \frac{5+3}{6+5} = \frac{8}{11}$$

## Practice for Skill—Subtracting Fractions

The denominator of a fraction tells you the size of the part. The numerator tells you how many parts are to be used.

When you subtract fractions, you subtract only the numerators. Why?

$\frac{7}{9} - \frac{5}{9} = ?$

7 ninths
− 5 ninths
2 ninths

$\frac{13}{16} - \frac{5}{16} = ?$

13 sixteenths
− 5 sixteenths
? sixteenths

Subtract these like fractions:

**1.** $\frac{8}{9} - \frac{1}{9}$ **2.** $\frac{9}{10} - \frac{7}{10}$ **3.** $\frac{11}{12} - \frac{7}{12}$ **4.** $\frac{5}{6} - \frac{1}{6}$

**5.** $\frac{7}{8} - \frac{5}{8}$ **6.** $\frac{5}{7} - \frac{2}{7}$ **7.** $\frac{7}{10} - \frac{3}{10}$ **8.** $\frac{7}{5} - \frac{2}{5}$ **9.** $\frac{11}{6} - \frac{5}{6}$ **10.** $\frac{6}{2} - \frac{2}{2}$

Subtract these unlike fractions:

| | a. | b. | c. | d. | e. | f. |
|---|---|---|---|---|---|---|
| **11.** | $\frac{2}{3} - \frac{2}{5}$ | $\frac{3}{4} - \frac{1}{5}$ | $\frac{5}{7} - \frac{2}{3}$ | $\frac{3}{10} - \frac{1}{4}$ | $\frac{4}{9} - \frac{1}{6}$ | $\frac{5}{8} - \frac{1}{3}$ |
| **12.** | $\frac{1}{3} - \frac{1}{4}$ | $\frac{1}{5} - \frac{3}{20}$ | $\frac{7}{12} - \frac{1}{2}$ | $\frac{3}{5} - \frac{1}{2}$ | $\frac{5}{7} - \frac{1}{2}$ | $\frac{3}{4} - \frac{2}{3}$ |

**13.** Why is it incorrect to do this?

$$\frac{7}{8} - \frac{2}{3} = \frac{7-2}{8-3} = \frac{5}{5} = 1$$

**14.** Why is it never correct to subtract the denominators in a subtraction exercise?

Tell first how you may be able to subtract these fractions, then do the work:

| | a. | b. | c. | d. | e. | f. |
|---|---|---|---|---|---|---|
| **15.** | $\frac{7}{8} - \frac{1}{2}$ | $\frac{1}{2} - \frac{3}{8}$ | $\frac{3}{4} - \frac{3}{8}$ | $\frac{1}{3} - \frac{1}{6}$ | $\frac{3}{6} - \frac{1}{2}$ | $\frac{7}{12} - \frac{1}{6}$ |
| **16.** | $\frac{2}{3} - \frac{3}{7}$ | $\frac{8}{9} - \frac{1}{2}$ | $\frac{5}{6} - \frac{3}{4}$ | $\frac{9}{8} - \frac{2}{3}$ | $\frac{11}{16} - \frac{2}{4}$ | $\frac{9}{10} - \frac{2}{3}$ |



## Sue Makes Her Own Greeting Cards

As soon as Sue decided to make her own greeting cards for the holiday season, she wanted to know how much material she would need. First, she got a card that was just the size she had in mind. Then, she measured it. She found after measuring carefully that it was $3\frac{1}{2}$ inches long. She wrote three and one half like this: $3\frac{1}{2}$. She saw that the 3 changed to halves gave 6 halves. $\frac{6}{2} + \frac{1}{2} = \frac{7}{2}$.

Then, Sue discovered something: The 2 in the denominator times the whole number, 3, told how many halves in 3. So, if she multiplied $2 \times 3$ and added the 1 in the numerator of the fraction $\frac{1}{2}$ to the result, this gave her the number of halves in $3\frac{1}{2}$. She changed the mixed number to an improper fraction.

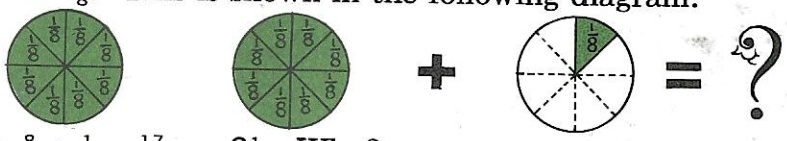
To change mixed numbers to improper fractions, you multiply the whole number by the denominator and add the numerator to the product, placing the result over the denominator.

Check the rule by changing these mixed numbers to improper fractions:

| | a. | b. | c. | d. | e. | f. |
|---|---|---|---|---|---|---|
| 1. | $2\frac{3}{4}$ | $1\frac{5}{8}$ | $7\frac{2}{3}$ | $3\frac{3}{8}$ | $4\frac{2}{7}$ | $5\frac{1}{2}$ |
| 2. | $9\frac{3}{5}$ | $7\frac{5}{8}$ | $6\frac{3}{5}$ | $4\frac{3}{4}$ | $6\frac{7}{8}$ | $7\frac{1}{8}$ |
| 3. | $7\frac{3}{4}$ | $5\frac{3}{8}$ | $3\frac{4}{5}$ | $7\frac{3}{4}$ | $8\frac{3}{6}$ | $12\frac{7}{8}$ |

## Further Understandings in Fractions

Add: $2 + \frac{1}{8} = ?$ You know that this gives the mixed number $2\frac{1}{8}$. This is shown in the following diagram:



$\frac{8}{8} + \frac{8}{8} + \frac{1}{8} = \frac{17}{8}$, or $2\frac{1}{8}$. Why?

How do you change a whole number to a fraction with a denominator of 8? Is the whole number $2 = \frac{16}{8}$? Explain.

How would you add $3 + \frac{3}{4}$ as fractions?

Are the following forms correct for adding 2 and $\frac{2}{5}$? Why?

**a.**

$$\begin{array}{r} 2 \\ +\ 0\frac{2}{5} \\ \hline 2\frac{2}{5} \end{array}$$

**b.**

$$\begin{array}{r} 2 \\ +\ 0\frac{2}{5} \\ \hline \end{array} = \begin{array}{r} \frac{10}{5} \\ +\ \frac{2}{5} \\ \hline \end{array} = \frac{12}{5} = 2\frac{2}{5}$$

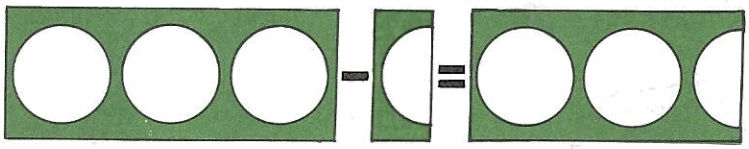
Add the following two ways:

| | | | | |
|---|---|---|---|---|
| **1.** | $10 + \frac{1}{6} = ?$ | $15 + \frac{2}{8} = ?$ | $25 + \frac{2}{3} = ?$ | $8 + \frac{2}{3} = ?$ |
| **2.** | $20 + \frac{5}{9} = ?$ | $11 + \frac{4}{6} = ?$ | $5 + \frac{2}{3} = ?$ | $18 + \frac{5}{4} = ?$ |
| **3.** | $33 + \frac{1}{3} = ?$ | $7 + \frac{13}{4} = ?$ | $3 + \frac{1}{12} = ?$ | $14 + \frac{7}{5} = ?$ |
| **4.** | $9 + \frac{4}{7} = ?$ | $12 + \frac{4}{10} = ?$ | $29 + \frac{3}{7} = ?$ | $8 + \frac{11}{9} = ?$ |

Find the sums:

| | **a.** | **b.** | **c.** | **d.** | **e.** | **f.** | **g.** |
|---|---|---|---|---|---|---|---|
| **5.** | $6\frac{1}{4}$ | $2\frac{1}{8}$ | $\frac{2}{3}$ | $3\frac{1}{2}$ | $7$ | $3\frac{1}{2}$ | $2\frac{1}{3}$ |
| | $8$ | $5$ | $\frac{5}{6}$ | $5$ | $2\frac{3}{4}$ | $1\frac{5}{16}$ | $\frac{5}{7}$ |
| | $7$ | $1\frac{3}{4}$ | $4$ | $7$ | $5$ | $6$ | $6$ |
| **6.** | $8$ | $3$ | $4$ | $\frac{5}{7}$ | $2$ | $6\frac{2}{3}$ | $8\frac{1}{4}$ |
| | $\frac{5}{9}$ | $\frac{2}{7}$ | $\frac{3}{8}$ | $6$ | $4\frac{1}{2}$ | $7$ | $3$ |
| **7.** | $3\frac{1}{2}$ | $6\frac{1}{3}$ | $8$ | $2\frac{1}{7}$ | $2\frac{1}{3}$ | $2\frac{7}{8}$ | $1\frac{5}{6}$ |
| | $5$ | $2\frac{1}{4}$ | $2\frac{1}{5}$ | $3\frac{1}{2}$ | $3\frac{1}{2}$ | $4\frac{1}{3}$ | $8\frac{1}{3}$ |
| | $2\frac{2}{3}$ | $5$ | $4\frac{1}{2}$ | $7$ | $5\frac{1}{5}$ | $5\frac{1}{6}$ | $1\frac{3}{4}$ |

Subtract $3 - \frac{1}{2}$:



**a.** $3 - 0\frac{1}{2} = 2\frac{2}{2} - 0\frac{1}{2}$

$2\frac{1}{2}$ Why?

You see in the diagram that you must take the $\frac{1}{2}$ from one of the whole ones. So, use 1 of the 3 wholes, and change it to halves for the subtraction. Why?

Another way is to change the 3 wholes into halves. You then have:

**b.** $3 - 0\frac{1}{2} = \frac{6}{2} - \frac{1}{2}$

$\frac{5}{2} = 2\frac{1}{2}$ Why?

How many halves in a whole? in 3 wholes? Tell why 3 becomes an improper fraction when it is changed to halves.

**1.** Subtract these by both methods:

| **a.** | **b.** | **c.** | **d.** | **e.** | **f.** | **g.** |
|---|---|---|---|---|---|---|
| $5 - 0\frac{1}{4}$ | $8 - 0\frac{2}{5}$ | $10 - 0\frac{2}{3}$ | $15 - 0\frac{4}{6}$ | $9 - 0\frac{6}{10}$ | $7 - 0\frac{5}{7}$ | $11 - 0\frac{3}{8}$ |

**2.** Can 3 be written as $\frac{3}{1}$? Does this show that the denominator of a fraction tells the number of parts there are in a single 1-unit thing? Explain.

**3.** How many thirds are there in 1? How many eighths?

**4.** Subtract orally, using 1 from the whole number for fractional parts:

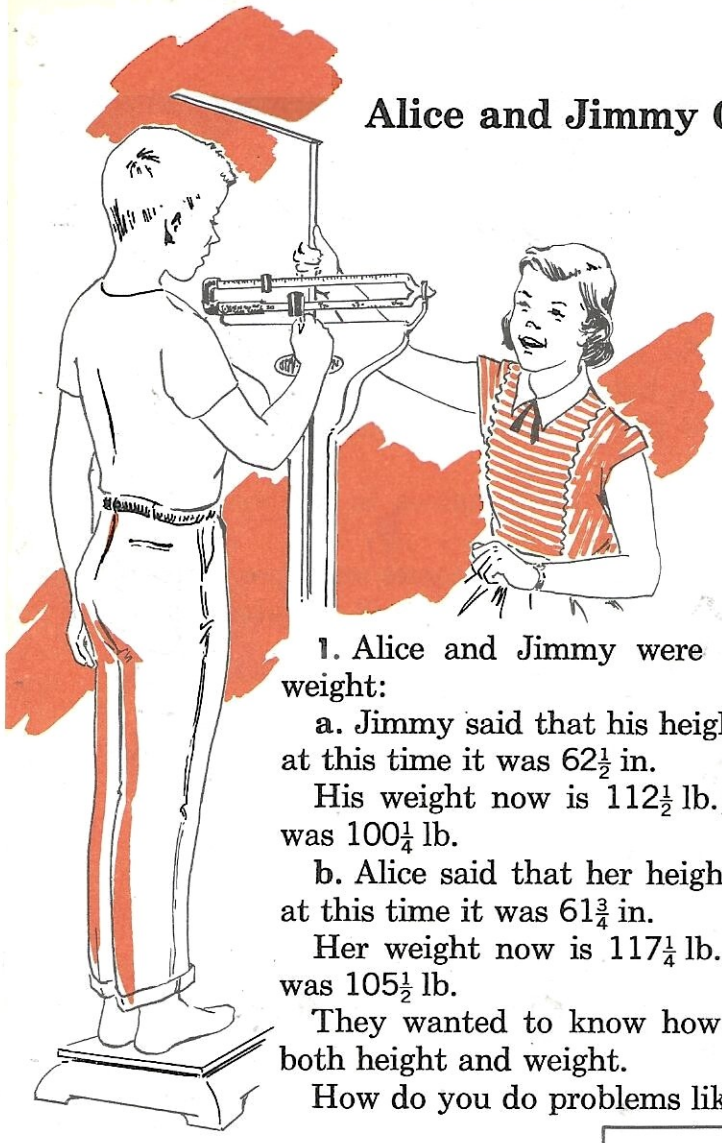
$5 - \frac{1}{2} = ?$ $7 - \frac{1}{4} = ?$ $6 - \frac{2}{3} = ?$ $4 - \frac{2}{5} = ?$ $8 - \frac{3}{4} = ?$

**5.** Which is more, $2\frac{8}{8}$ or $3\frac{1}{8}$? How much more?

**6.** Which method, **a** or **b** given above, do you like better for subtracting a fraction from a whole number? Why?

**7.** Subtract:

| **a.** | **b.** | **c.** | **d.** | **e.** | **f.** | **g.** |
|---|---|---|---|---|---|---|
| $8 - \frac{2}{3}$ | $4 - \frac{4}{5}$ | $3 - \frac{2}{9}$ | $5 - \frac{5}{16}$ | $1 - \frac{7}{9}$ | $2 - \frac{5}{8}$ | $11 - \frac{5}{6}$ |



## Alice and Jimmy Check Their Heights

Boys and girls of your age are often interested in knowing if their heights and weights are about average for their age.

Do you know how much weight you have gained and how tall you have grown in the last year?

**1.** Alice and Jimmy were checking their height and weight:

**a.** Jimmy said that his height now is $64\frac{3}{4}$ in.; last year at this time it was $62\frac{1}{2}$ in.

His weight now is $112\frac{1}{2}$ lb.; last year at this time it was $100\frac{1}{4}$ lb.

**b.** Alice said that her height now is $63\frac{1}{4}$ in.; last year at this time it was $61\frac{3}{4}$ in.

Her weight now is $117\frac{1}{4}$ lb.; last year at this time it was $105\frac{1}{2}$ lb.

They wanted to know how much each had gained in both height and weight.

How do you do problems like theirs?

What kinds of numbers are these? Are they three-place numbers? There are three units in each height —tens, ones, and fractional parts of ones.

| Alice | Jimmy |
|---|---|
| height<br>$63\frac{1}{4}$ in.<br>$-\ 61\frac{3}{4}$ in. | height<br>$64\frac{3}{4}$ in.<br>$-\ 62\frac{1}{2}$ in. |
| weight<br>$117\frac{1}{4}$ lb.<br>$-\ 105\frac{1}{2}$ lb. | weight<br>$112\frac{1}{2}$ lb.<br>$-\ 110\frac{1}{4}$ lb. |

## Solving Alice's and Jimmy's Problems

Jimmy's height:

$$\begin{array}{r} 64\frac{3}{4} \\ -\ 62\frac{1}{2} \end{array} = \begin{array}{r} 64\frac{3}{4} \\ -\ 62\frac{2}{4} \\ \hline 2\frac{1}{4} \end{array}$$

Subtract the fractional parts first: $\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$. Why? Write $\frac{1}{4}$ in the answer so that it is in the column with the fractional parts. $64 - 62 = 2$. Write 2 in ones place in the answer.

Jimmy gained $2\frac{1}{4}$ in. in height.

Jimmy's weight:

$$\begin{array}{r} 112\frac{1}{2} \\ -\ 100\frac{1}{4} \\ \hline ? \end{array}$$

$\frac{1}{2} - \frac{1}{4} = \frac{?}{4} - \frac{?}{4} = ?$ Why?
$112 - 100 = ?$

Jimmy gained ? lb. in weight.

Alice's height:

$$\begin{array}{r} 63\frac{1}{4} \\ -\ 61\frac{3}{4} \\ \hline ? \end{array} = \begin{array}{r} 62\frac{5}{4} \\ 61\frac{3}{4} \\ \hline ? \end{array}$$

You cannot subtract $\frac{3}{4}$ from $\frac{1}{4}$. Why? You must change one of the whole pounds to fractional parts of a pound. Change one of the whole ones of 63 to $\frac{4}{4}$. Do you have to add $\frac{1}{4}$ to $\frac{4}{4}$? Finish the problem.

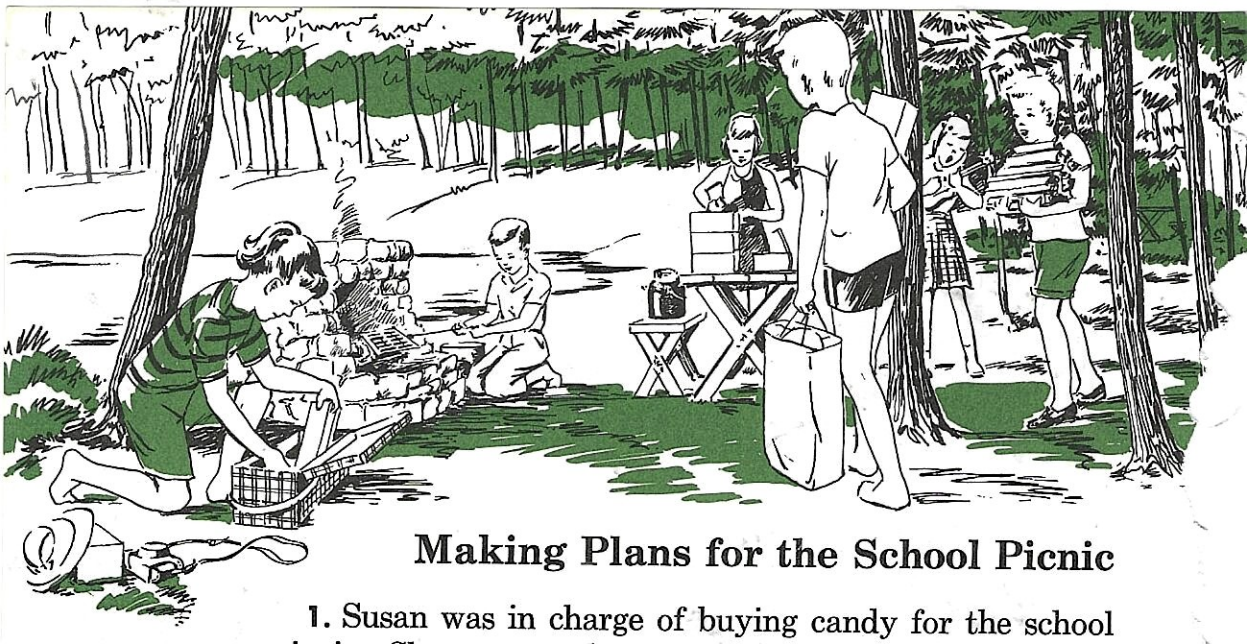
Alice's weight:

$$\begin{array}{r} 117\frac{1}{4} \\ -\ 105\frac{1}{2} \end{array} = \begin{array}{r} 116\frac{?}{4} \\ -\ 105\frac{?}{4} \end{array}$$

Do you have to change a whole one to fractional parts for this problem? Finish the work.

Subtract:

1. $\begin{array}{r} 59\frac{8}{10} \\ -\ 53\frac{3}{10} \end{array}$
2. $\begin{array}{r} 67\frac{4}{10} \\ -\ 64\frac{7}{10} \end{array}$
3. $\begin{array}{r} 65\frac{3}{4} \\ -\ 62\frac{1}{6} \end{array}$
4. $\begin{array}{r} 110\frac{1}{8} \\ -\ 105\frac{3}{4} \end{array}$
5. $\begin{array}{r} 18 \\ -\ 2\frac{3}{5} \end{array}$



## Making Plans for the School Picnic

**1.** Susan was in charge of buying candy for the school picnic. She went to the store and bought $3\frac{1}{2}$ lb. of chocolates, $5\frac{1}{2}$ lb. of peanut brittle, and $4\frac{1}{2}$ lb. of butterscotch candy. How many pounds of candy did Susan buy in all?

**2.** Jane bought rolls at the bakery for the picnic. She bought $4\frac{1}{3}$ dozen hard rolls, $4\frac{1}{4}$ dozen pecan rolls, and $4\frac{1}{2}$ dozen plain rolls. How many dozen rolls did Jane buy in all?

**3.** Betty bought $4\frac{1}{2}$ lb. of hot dogs from Mr. Brown's store. Ed's father gave him 5 lb. of hamburger from his store, and Mary and Alma brought $2\frac{1}{2}$ lb. of hot dogs each. How many pounds of meat did the children have altogether?

**4.** Betty needed $1\frac{1}{2}$ doz. eggs for custards, $1\frac{1}{2}$ doz. for cakes, and $1\frac{3}{4}$ doz. for egg salad. How many dozen eggs did Betty need?

**5.** Joe, Betty, and Ann made sandwiches. Ann made $1\frac{2}{3}$ doz., Joe $2\frac{1}{4}$ doz., and Betty made only $\frac{5}{6}$ doz. Find the number of dozen the 3 children made.

**6.** Three children were responsible for making lemonade. Bob made $3\frac{1}{2}$ gal., Helen, $2\frac{3}{4}$ gal., and Bill, 4 gal. They needed 15 gallons in all. How many more gallons were needed?

## Problems for Practice

**1.** Find the difference between $\frac{1}{8}$ of a yard and $\frac{1}{2}$ of a yard of material.

**2.** Joan had $\frac{3}{4}$ of a yard of scrap cloth and $\frac{2}{3}$ of a yard of new cloth. How much did she have altogether?

**3.** Grace bought $\frac{3}{4}$ of a yard of organdy and $\frac{1}{2}$ of a yard of broadcloth. How much material did she buy? She bought more of which kind of material? How much more?

**4.** June bought $\frac{3}{4}$ of a yard of cotton to make an apron. She found that she needed only $\frac{1}{2}$ of a yard. How much material did she have left?

**5.** Find how much $\frac{1}{3}$ and $\frac{1}{2}$ of a yard are altogether.

**6.** $\frac{1}{2}$ yd. is how much more than $\frac{1}{8}$ of a yard?

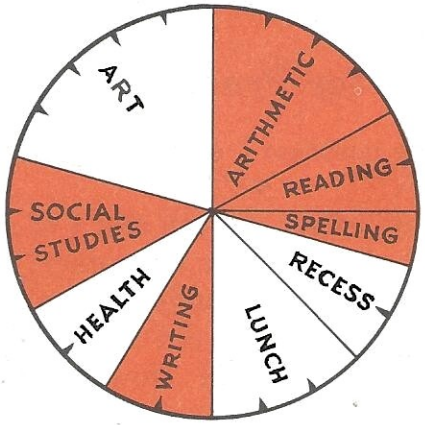
**7.** If Mary bought $\frac{3}{4}$ of a pound of chocolates and $\frac{1}{2}$ of a pound of jelly beans, how many pounds of candy did she buy?

**8.** Bill and Dave were making a sailboat. They needed $4\frac{3}{8}$ yards of canvas for one sail and $1\frac{1}{4}$ yards for the other. How much canvas did they need in all?

**9.** Mary helped her mother measure their kitchen windows for curtains. The large window was $1\frac{1}{3}$ yards long while the small window was $1\frac{1}{6}$ yards long. What length of curtain material will be long enough to cover both windows? How much longer is the large window than the smaller one?

**10.** Chris and Sue each bought a piece of cloth for their play room. If Chris' cloth was $1\frac{5}{8}$ yards long and Sue's was $2\frac{3}{4}$ feet long, how long were they together? Whose piece was the larger? How much larger?

**11.** Chris and Sue bought $1\frac{5}{8}$ lb. of cashew nuts and $2\frac{3}{4}$ lb. of peanuts for a party. Their mother bought $2\frac{1}{4}$ lb. of chocolates and $2\frac{1}{16}$ lb. of hard candy. How many pounds of nuts were there for the party? How much candy? Which was the greater amount, the nuts or the candy? How much greater?




## Phil's School Day

Lunch—45 min.
Recess— 30 min.
Arithmetic—60 min.
Arts—75 min.
Social Studies—45 min.
Reading—30 min.
Spelling—15 min.
Health—30 min.
Writing—30 min.

Phil lives in a town where the school day lasts from 9 A.M. to 3 P.M. This circle represents the number of hours that he is at school. If the whole circle is divided into parts, each to represent the amount of time that he spends in his different activities, this is called a **circle graph.**

**1.** What are Phil's different activities during a school day?

**2.** Find the fractional part of the circle that represents the time he spends in an arithmetic period.

**3.** Find the fractional parts of the total time spent in health, spelling, and lunch periods.

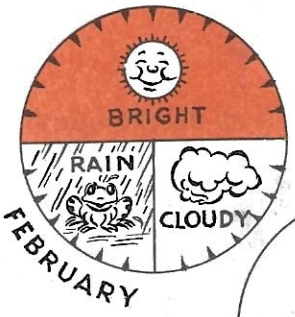
**4.** Compare the fractional parts that Phil spends in arts and social studies. Which is the greater? How much greater?

**5.** Add all of the times for the different periods. Does the sum equal the total time from the opening to the close of school?

**6.** Find the remaining fractional parts of Phil's school day which are not given in the above problems. Find the total of all of the fractional parts. What does the sum mean?

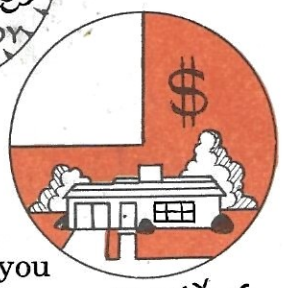
**7.** Make a collection of circle graphs (you can find them in newspapers and magazines). Choose only those which contain information that is interesting to you. Tell the stories to the class.

## Making Graphs




**1.** Draw a circle and divide it to show the parts of the month of February where 14 days were bright, 7 days were rainy, and 7 days were cloudy.

**2.** Make a circle graph which shows that your family used $\frac{3}{4}$ of their savings to build a new home.

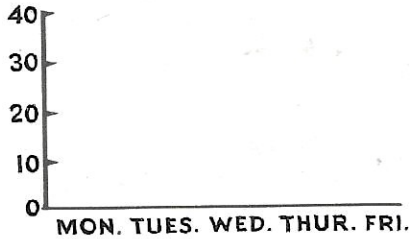



**3.** Show on a circle graph the part of 24 hours that you spend in school if the opening bell rings at 8:30 A.M., you have from 11:30 A.M. to 12:30 P.M. for lunch, and school closes at 3:30 P.M.

**4.** Some clocks and watches today do not show the hour and minute spaces on their faces. You tell time by the position of the hands only. What figuring do you have to do in order to tell the time on such time pieces? Tell how this is something like making a circle graph.

**5.** If you were drawing a circle graph with parts like the positions of the hour hand for the even hours on a clock face, how many parts would there be? If one part of this graph stands for 15 things, what is the value of the whole circle? of the half-circle?

**6.** Draw a bar graph to show the attendance in your class for the last 5 days. Put the names of the days at the bottom and the number of pupils in the column on the left.




**7.** How would you draw a bar graph to show the rainy days on each of the last 30 school days?

**8.** Draw a bar graph to show the days on which you had fruit juice for breakfast for a week.

## Unit Review

Add as mixed numbers:

| 1. | 2. | 3. | 4. | 5. |
|---|---|---|---|---|
| $9\frac{2}{6}$ | $6\frac{1}{4}$ | $4\frac{2}{5}$ | $5\frac{3}{10}$ | $\frac{5}{8}$ |
| $3\frac{4}{6}$ | $2\frac{2}{3}$ | $1\frac{1}{2}$ | $4\frac{3}{10}$ | $\frac{2}{3}$ |

Add after changing mixed numbers to improper fractions:

| 6. | 7. | 8. | 9. | 10. |
|---|---|---|---|---|
| $5\frac{3}{4}$ | $8\frac{4}{5}$ | $10\frac{2}{7}$ | $12\frac{1}{4}$ | $\frac{3}{4}$ |
| $1\frac{1}{4}$ | $4\frac{1}{5}$ | $6\frac{5}{7}$ | $5\frac{2}{3}$ | $\frac{5}{6}$ |

Change to lowest terms:

| 11. | 12. | 13. | 14. | 15. |
|---|---|---|---|---|
| $\frac{15}{24}$ | $\frac{8}{12}$ | $\frac{21}{35}$ | $\frac{9}{15}$ | $\frac{11}{13}$ |

Subtract as mixed numbers:

| 16. | 17. | 18. | 19. |
|---|---|---|---|
| $6\frac{1}{8}$ | $10\frac{2}{3}$ | $15\frac{3}{7}$ | $8\frac{2}{3}$ |
| $1\frac{3}{4}$ | $2\frac{2}{5}$ | $3\frac{2}{3}$ | $4\frac{1}{2}$ |

Subtract after changing mixed numbers to improper fractions:

| 20. | 21. | 22. | 23. |
|---|---|---|---|
| $4\frac{3}{10}$ | $9\frac{2}{9}$ | $14\frac{4}{5}$ | $12\frac{2}{3}$ |
| $1\frac{2}{3}$ | $2\frac{1}{2}$ | $3\frac{1}{4}$ | $4\frac{5}{8}$ |

Change to mixed numbers:

**24.** $\frac{11}{4}$ $\frac{13}{3}$ $\frac{25}{7}$ $\frac{18}{5}$ $\frac{9}{4}$ $\frac{13}{2}$ $\frac{15}{8}$ $\frac{11}{3}$

Problem solving:

**1.** Ann was in charge of the church party. She decided to decorate with red, white, and blue crepe paper. She bought the paper in rolls of 30 yards each. If she used $27\frac{3}{4}$ yards of blue, $14\frac{3}{8}$ yards of red, and $24\frac{7}{8}$ yards of white, how many yards of crepe paper were used? Was there any paper left from each roll? How much of each kind?

**2.** Ann used the same color scheme for food: $2\frac{3}{4}$ dozen cup cakes were frosted red, $2\frac{1}{2}$ dozen were frosted white, and $2\frac{2}{3}$ dozen were frosted blue. She had 8 doz. cakes in all, so she decided to leave the others plain. Find how many dozen were not frosted.

## Unit Test

**1.** Judy needed material to make some dresses for her sister's doll. She needed $\frac{1}{4}$ yd. of woolen cloth, $\frac{1}{5}$ yd. of felt, and $\frac{1}{2}$ yd. of cotton cloth. How much material did she need altogether?

**2.** Alice ordered $2\frac{1}{3}$ pounds of ground beef and $1\frac{1}{6}$ pounds of ground pork for meatloaf. What was the total weight of the meat for the loaf?

**3.** George was testing the speedometer on his new bike. He knew it was $2\frac{3}{10}$ mi. to Ted's house and that Ted lived $\frac{3}{5}$ mi. on the other side of the movie house. What distance should his speedometer show after riding from his house to the movie house?

**4.** Jean wanted to make a stew for dinner. She bought $1\frac{3}{4}$ pounds of pork and $1\frac{3}{8}$ pounds of veal. How much meat did she buy? How much more pork than veal did she buy?

**5.** Change $1\frac{7}{8}$ to an improper fraction.

**6.** Change $\frac{16}{5}$ to a mixed number.

**7.** Change each of the following fractions to its lowest terms:

$\frac{9}{15}$ $\frac{16}{24}$ $\frac{12}{32}$ $\frac{40}{64}$ $\frac{25}{35}$ $\frac{16}{22}$ $\frac{10}{16}$ $\frac{15}{27}$

**8.** Tell the real meaning of changing a fraction to its lowest terms; of changing a fraction to higher terms.

**9.** What is the meaning of a common denominator (use measuring idea)?

**10.** What is the meaning of the L.C.D. of two or more fractions?

Find the answers as indicated:

**11.** $8\frac{5}{8} + 0\frac{1}{6}$ **12.** $\frac{5}{6} + \frac{3}{4}$ **13.** $6\frac{3}{5} + 2\frac{1}{2}$ **14.** $\frac{7}{8} - \frac{2}{3}$ **15.** $3 - 1\frac{3}{4}$ **16.** $8\frac{1}{2} - 2\frac{2}{3}$

# UNIT 7

# Using Your Spare Time



Here are some statements that some children made when they were asked the question: How do you use your spare time?

"I like to write to children in other countries. I mail the letters myself. It costs me 15¢ a half-ounce for postage and takes four or five days for delivery. I write to a girl in Finland and a boy in France."

"I like to sew clothes, and decorate things, like putting coins, beads, different colored stones, and rhinestones on scarfs and hats."

"My hobby is collecting records, especially hit tunes. It started when I received a $5.00 record certificate from my aunt. Now I have about fifty records."

"I have started to collect stamps from all over the world. I have two from Holland, four from Italy, one from Germany and one from Greenland."

"I collect dolls from different countries, popular records, small perfume bottles, and stamps."

"My hobby is tropical fish. I have all kinds of them. I breed them and sell them, too. I have six tanks, with air and heating controls in each tank."

Some other hobbies that boys and girls of your age have are: handicrafts, cooking, sewing, bicycling, roller skating, and painting.

Cliff is trying to cut 6 boards, $1\frac{3}{4}$ feet long, from a board 8 feet long. Can he do it? How many pieces of that length can he cut?

Marian's doll is wearing her new dress. Marian made the dress from $\frac{5}{8}$ yard of material. How much material would she need to make 3 dresses?

Sam takes and develops his own pictures. He plans to enlarge some of the best ones. The sheets of paper for enlargements are $8\frac{1}{2}$ inches wide. Would it be more economical for his enlargements to be $3\frac{1}{4}$ inches wide or $4\frac{1}{4}$ inches wide?





## Jay's Hobby Is Bicycle Riding

Jay lives in the country. Riding his bike is his big interest. He is trying to increase his speed as he rides up and down the lane to the main highway. The lane is $\frac{5}{8}$ mile long, and Jay uses his wrist watch to measure his time.



$\frac{5}{8}$ mile



What does $\frac{5}{8}$ mile mean? Jay knows he can think of this distance in 2 different ways. Here they are:

**1.** He can think of the mile as a whole. If he divides it into 8 equal parts and uses 5 of these parts, he will be riding $\frac{5}{8}$ of a whole, or $\frac{5}{8}$ mi.



**2.** He can think of a distance of $\frac{1}{8}$ mi. as a unit. If he rides 5 of them, or $5 \times \frac{1}{8}$ mi., he will cover a distance of $\frac{5}{8}$ mi.

You see that both ideas lead to the same result. The difference is that in **1** the part $\frac{1}{8}$ is not named (the $\frac{5}{8}$ is 5 out of 8 equal parts of a whole), while in **2** the part is named $\frac{1}{8}$ mi., being a small unit that is found by dividing a larger whole into equal parts.

When you use fractional measurements in your work, method **2** is the way to think: $\frac{3}{4}$ yd. is 3 of the $\frac{1}{4}$ yd. units, and $\frac{5}{12}$ ft. is 5 of the $\frac{1}{12}$ units (inch) = 5 in. When you use a fraction like $\frac{5}{6}$, it means that a whole has been divided into 6 equal parts and you are using 5 of them.

Here are some statements that some children made when they were asked the question: How do you use your spare time?

"I like to write to children in other countries. I mail the letters myself. It costs me 15¢ a half-ounce for postage and takes four or five days for delivery. I write to a girl in Finland and a boy in France."

"I like to sew clothes, and decorate things, like putting coins, beads, different colored stones, and rhinestones on scarfs and hats."

"My hobby is collecting records, especially hit tunes. It started when I received a $5.00 record certificate from my aunt. Now I have about fifty records."

"I have started to collect stamps from all over the world. I have two from Holland, four from Italy, one from Germany and one from Greenland."

"I collect dolls from different countries, popular records, small perfume bottles, and stamps."

"My hobby is tropical fish. I have all kinds of them. I breed them and sell them, too. I have six tanks, with air and heating controls in each tank."

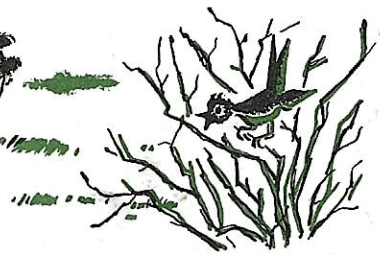
Some other hobbies that boys and girls of your age have are: handicrafts, cooking, sewing, bicycling, roller skating, and painting.

Cliff is trying to cut 6 boards, $1\frac{3}{4}$ feet long, from a board 8 feet long. Can he do it? How many pieces of that length can he cut?

Marian's doll is wearing her new dress. Marian made the dress from $\frac{5}{8}$ yard of material. How much material would she need to make 3 dresses?

Sam takes and develops his own pictures. He plans to enlarge some of the best ones. The sheets of paper for enlargements are $8\frac{1}{2}$ inches wide. Would it be more economical for his enlargements to be $3\frac{1}{4}$ inches wide or $4\frac{1}{4}$ inches wide?





## Jay's Hobby Is Bicycle Riding

Jay lives in the country. Riding his bike is his big interest. He is trying to increase his speed as he rides up and down the lane to the main highway. The lane is $\frac{5}{8}$ mile long, and Jay uses his wrist watch to measure his time.



$\frac{5}{8}$ mile



What does $\frac{5}{8}$ mile mean? Jay knows he can think of this distance in 2 different ways. Here they are:

**1.** He can think of the mile as a whole. If he divides it into 8 equal parts and uses 5 of these parts, he will be riding $\frac{5}{8}$ of a whole, or $\frac{5}{8}$ mi.



**2.** He can think of a distance of $\frac{1}{8}$ mi. as a unit. If he rides 5 of them, or $5 \times \frac{1}{8}$ mi., he will cover a distance of $\frac{5}{8}$ mi.

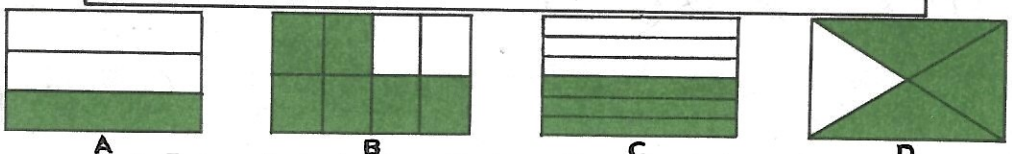
You see that both ideas lead to the same result. The difference is that in **1** the part $\frac{1}{8}$ is not named (the $\frac{5}{8}$ is 5 out of 8 equal parts of a whole), while in **2** the part is named $\frac{1}{8}$ mi., being a small unit that is found by dividing a larger whole into equal parts.

When you use fractional measurements in your work, method **2** is the way to think: $\frac{3}{4}$ yd. is 3 of the $\frac{1}{4}$ yd. units, and $\frac{5}{12}$ ft. is 5 of the $\frac{1}{12}$ units (inch) = 5 in. When you use a fraction like $\frac{5}{6}$, it means that a whole has been divided into 6 equal parts and you are using 5 of them.

## Check Your Understanding

Explain the statements in this box:

> The denominator of a fraction tells the size of the part of a whole or the size of the fractional unit. The numerator of a fraction tells how many of the units or parts there are in use.




**1.** Into what sized parts has diagram A been divided? Write the fraction that tells the part that has not been colored.

**2.** Which of the above diagrams has $\frac{3}{4}$ of it colored?

**3.** Give two fractions which tell the colored part of diagram C.

**4.** Explain how $\frac{2}{3}$ yd. may be thought of as 2 of the units of the $\frac{1}{3}$ yd. size.

**5.** Explain the meaning of the value $\frac{5}{6}$.

**6.** Explain each of these fractions as representing a certain number of units of a given size:

$\frac{5}{16}$ lb. $\frac{7}{12}$ ft. $\frac{1}{6}$ yd. $\frac{2}{3}$ yd. $\frac{7}{8}$ lb. $\$\frac{3}{5}$

**7.** What are the number of $\frac{1}{4}$ units in $\frac{1}{2}$ mi.? in $\frac{6}{8}$ mi.?

**8.** What is meant when you speak of a unit of measure?

**9.** Tell what two smaller units of the same size give a unit of $\frac{1}{4}$ lb.

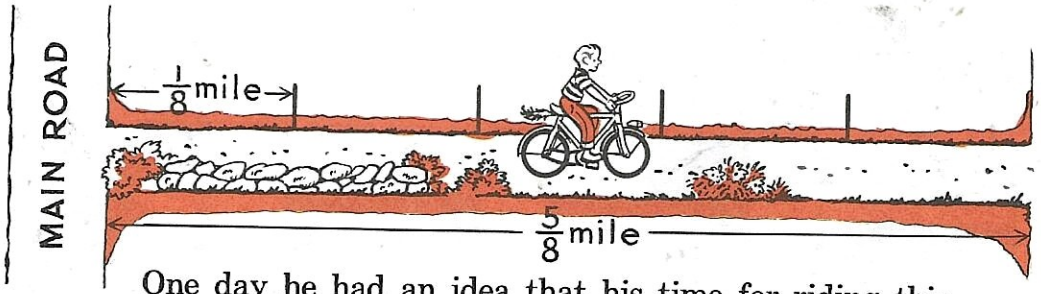
**10.** When could you think of $\frac{5}{16}$ as a group of units, and when as a part of a whole?

**11.** In what way is $\frac{5}{4}$ mi. the result of taking a unit of measure 5 times? Why does $\frac{5}{4}$ mi. = $1\frac{1}{4}$ mi.?

**12.** If you are cooking and the recipe calls for $\frac{1}{2}$ tsp. of salt, do you think of the $\frac{1}{2}$ as 1 of the 2 equal parts of a whole teaspoon, 2 of the units of $\frac{1}{4}$ tsp., or as a unit within itself?

## Jay's Rate of Riding

Jay rides the full length of his lane frequently. He thinks of the $\frac{5}{8}$ mi. as a distance, something very definite. It is a whole in his mind.




One day he had an idea that his time for riding this distance could be used to give him the time needed for other distances: the $\frac{5}{8}$ mi. is a value made up of units within itself. That's how it came to be $\frac{5}{8}$ mi., being 5 parts of $\frac{1}{8}$ mi. each.

His best time so far is 2 minutes or 120 seconds for the distance. Then, there being 5 units of the distance $\frac{1}{8}$ mi., he figured that $5\overline{)120}$ sec. would tell him his rate for every unit: $5\overline{)120} = 24$ sec. Why? Was he correct in his figuring?

**1.** Explain how $\frac{7}{10}$ mi. may be considered as a whole. What is the size of the part?

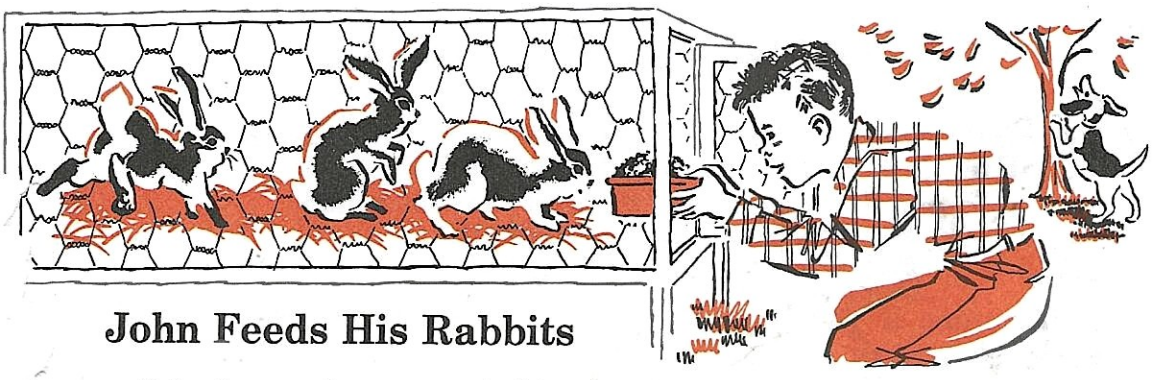
**2.** How could $\frac{9}{16}$ mi. be a grouping of parts?

**3.** At Jay's rate of riding, how long would it take him to ride $\frac{3}{8}$ mi.? $\frac{1}{2}$ mi.? $\frac{7}{8}$ mi.? $1\frac{1}{2}$ mi.? ($1\frac{1}{2}$ mi. = $\frac{12}{8}$ mi.)

**4.** One day, when Jay was going on a longer trip, he took candy bars for his lunch. He took all he had, 5 chocolate bars that were left from a box that originally contained a dozen bars. How can $\frac{5}{12}$ be the fraction that shows the chocolate bars he took?

**5.** If Jay could ride his bike at the rate of 24 sec. for each $\frac{1}{8}$ mi., how long would it take him to ride a quarter-mile? three quarter-miles?

**6.** How many $\frac{1}{8}$'s are there in $\frac{7}{8}$? in $\frac{15}{8}$? in $\frac{24}{8}$?



## John Feeds His Rabbits

John has to buy several things for his pets. The most expensive item that he buys regularly is food.

The rabbits eat $\frac{3}{4}$ lb. of shelled corn a week. John gives it to them in $\frac{1}{4}$ lb. parts, which is the unit in the value $\frac{3}{4}$ lb. If he wants to figure out how much they eat in 5 weeks, will the unit in $\frac{3}{4}$ lb. help him to find the answer? How? Can he use the $\frac{3}{4}$ lb. as a whole? Explain these figures:

1st week 2nd week 3rd week 4th week 5th week

$$\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{15}{4} = 3\frac{3}{4} \text{ lb.}$$

Here $\frac{3}{4}$ is added five times. Do you remember the short cut for adding equal units? Multiplication is the short process of addition.

$\frac{3}{4}$ can be used as a unit or whole. That is, $\frac{3}{4}$ is the whole of the corn that the rabbits eat in one week. Since you want to know the amount they will eat in 5 weeks, you can multiply $5 \times \frac{3}{4}$.

There is an important thing to know in multiplying a fraction by a whole number. You may see it if you notice which number in each fraction was used when you added.

In the $\frac{3}{4}$ values above, the 3 is the how-many of the units of the one-fourth size $(\frac{1}{4})$. Then, rather than add five 3's, you can multiply 3 by 5.

$\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$ or $5 \times \frac{3}{4} = \frac{5 \times 3}{4} = \frac{15}{4} = 3\frac{3}{4}$

## Checking Your Understanding

When adding fractions, the numerators (numbers that tell how many) are added. The denominators (numbers that tell the size of the part) are not added.

$$\frac{2}{9}+\frac{2}{9}+\frac{2}{9}=\frac{2+2+2}{9}=\frac{6}{9}$$

Therefore, if you have a group of like fractions (denominators alike) with the same numerators, and wish to find their total, you can multiply one of the numerators by the number of them.

Thus $3\times\frac{2}{9}=\frac{3\times 2}{9}=\frac{6}{9}=\frac{2}{3}$ Why?

**1.** How can the measurement $\frac{3}{4}$ lb. be considered as the result of a multiplication?

**2.** Multiply:

$7\times\frac{1}{8}$ in. $\quad 7\times\frac{2}{3}$ yd. $\quad 4\times\frac{3}{4}$ mi. $\quad 5\times\frac{1}{2}$ doz. $\quad 5\times\frac{3}{4}$ in.

$5\times\frac{1}{4}$ mi. $\quad 4\times\frac{2}{5}$ sec. $\quad 3\times\frac{3}{4}$ lb. $\quad 8\times\frac{2}{3}$ doz. $\quad 5\times\frac{3}{4}$ hr.

**3.** Tell which of these items can be written as multiplications, then find the answers:

$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=?$ $\qquad \frac{1}{4}+\frac{2}{3}+\frac{2}{5}+\frac{1}{2}=?$

$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=?$ $\qquad \frac{1}{2}+\frac{1}{3}+\frac{2}{5}+\frac{1}{4}=?$

**4.** Tell how you know that $15\times\frac{1}{4}=\frac{15}{4}=3+3\times\frac{1}{4}$.

**5.** If John's rabbits eat $\frac{1}{2}$ head of lettuce each day, how many heads will they eat in a week?

**6.** When you are measuring something with a given unit and find that you have a part left over which is less than a whole unit, you must use a part of that unit to measure the remaining part. Tell how many of the smaller units are found in the measurement $5\frac{4}{5}$ sec.

**7.** Explain the following ways of multiplying $2\times 1\frac{2}{9}$:

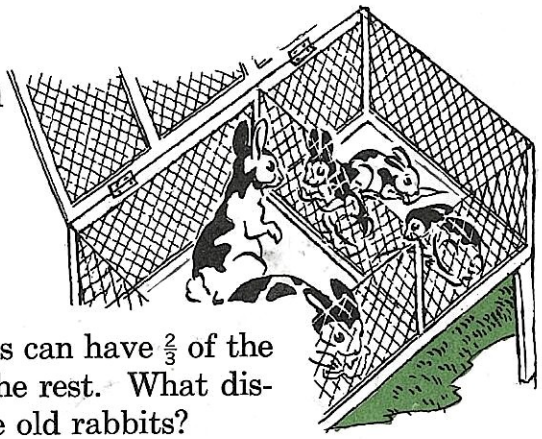
$$\begin{array}{r} 1\frac{2}{9} \\ \times\ 2 \\ \hline 2\frac{4}{9} \end{array} \qquad \begin{array}{l} 2\times\frac{2}{9}=\frac{4}{9} \\ 2\times 1=2 \end{array}$$

$$\begin{array}{r} 1\frac{2}{9} \\ \times\ 2 \\ \hline 2\frac{4}{9} \end{array} \qquad \begin{array}{l} 1\frac{2}{9}=\frac{11}{9} \\ 2\times\frac{11}{9}=\frac{22}{9}=2\frac{4}{9} \end{array}$$

**8.** Multiply:

$4\times\frac{2}{3}$ $\quad 3\times\frac{4}{7}$ $\quad 5\times\frac{5}{6}$ $\quad 9\times\frac{3}{4}$ $\quad 3\times\frac{3}{5}$ $\quad 2\times\frac{2}{7}$

## John Builds an Exercise Pen



John bought enough wire to make a pen for his rabbits. It is to be 12 ft. long and 8 ft. wide. He is going to make a partition so that the old rabbits can have $\frac{2}{3}$ of the pen and the young rabbits can have the rest. What distance must the partition cut off for the old rabbits?

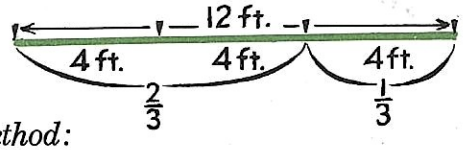
You want to find $\frac{2}{3}$ of the length, so you say $\frac{2}{3}$ of 12. It can be written $\frac{2}{3} \times 12$, because "of" means "times."

*First Method:*

$\frac{2}{3}$ is 2 of the $\frac{1}{3}$ parts. $\frac{1}{3}$ is 1 of the 3 equal parts of a whole.

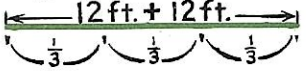
$\frac{1}{3}$ part of 12 is $3\overline{)12} = 4$; 2 parts (or $\frac{2}{3}$) $= 2 \times 4 = 8$

Therefore, $\frac{2}{3} \times 12$ ft. $= 8$ ft.



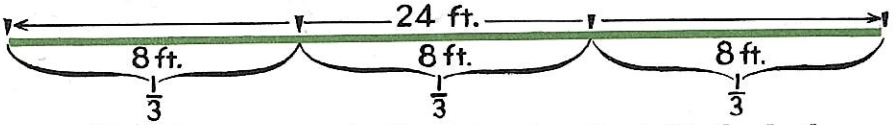

*Second Method:*

Since $\frac{2}{3}$ of a whole means 2 of the $\frac{1}{3}$ parts, you can take 2 of the wholes and then find $\frac{1}{3}$ of the result:

$\frac{1}{3}$ of 2 wholes =  gives

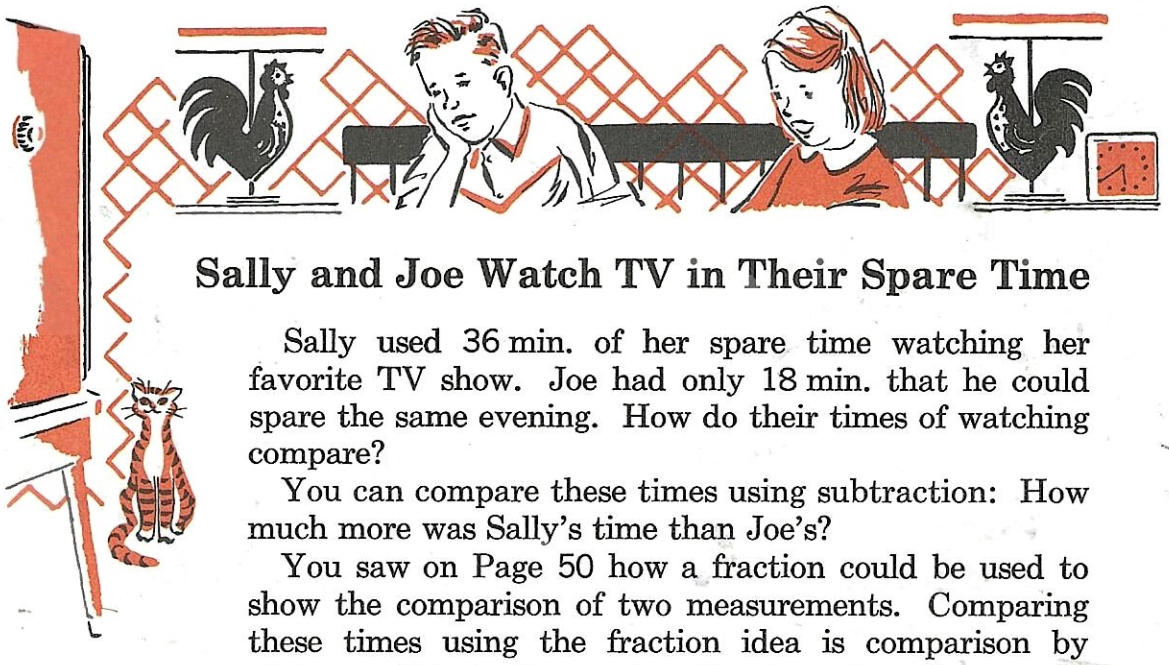
$\frac{1}{3}$ of 24 ft., or $3\overline{)24} = 8$ ft.

So, you can write $\frac{2}{3} \times 12$ ft. as $\frac{2 \times 12 \text{ ft.}}{3} = \frac{24 \text{ ft.}}{3} = 8$ ft.




**1.** Find the answers to these by the First Method, then check them by the Second Method:

| | | | |
|---|---|---|---|
| $\frac{3}{4}$ of 12 ft. | $\frac{2}{3}$ of 15 yd. | $\frac{2}{3} \times 9$ hr. | $\frac{5}{6} \times 30$ min. |
| $\frac{3}{8}$ of 16 rd. | $\frac{2}{3} \times 6$ gal. | $\frac{1}{4}$ of 8 bu. | $\frac{3}{4}$ of 16 T. |



## Sally and Joe Watch TV in Their Spare Time

Sally used 36 min. of her spare time watching her favorite TV show. Joe had only 18 min. that he could spare the same evening. How do their times of watching compare?

You can compare these times using subtraction: How much more was Sally's time than Joe's?

You saw on Page 50 how a fraction could be used to show the comparison of two measurements. Comparing these times using the fraction idea is comparison by division. Why? Comparing 36 min. with 18 min. you have 36 min. ÷ 18 min. or:

$$\frac{36 \text{ min.}}{18 \text{ min.}} = 2$$

This means that 36 min. is two groups of 18 min. In other words, Sally's time was twice Joe's time. $2 \times 18 = 36$

If you compare 18 min. with 36 min., you have 18 min. ÷ 36 min. or:

$$\frac{18 \text{ min.}}{36 \text{ min.}} = \frac{1}{2}$$

This means that 18 min. is $\frac{1}{2}$ of the group of 36 min. or Joe's time is $\frac{1}{2}$ of Sally's. $\frac{1}{2} \times 36 = 18$

When you compare two numbers by the division process, you always get a number that can be used as a multiplier of the second of the two numbers to give the first one. In other words, when comparing two numbers by division, *the quotient times the divisor will always give the dividend.*

The quotient or multiplier may be a whole number, a proper fraction, or an improper fraction.

Look at the comparisons in these examples:

**a.** 48 in. compared with 24 in. = $48 \div 24 = 2$.

This means that 2 groups of 24 in. = 48 in. (48 is twice as much as 24.)

**b.** 9 compared with 18 = $9 \div 18 = \frac{1}{2}$.

This means that $\frac{1}{2} \times 18 = 9$. (9 is one-half as much as 18.)

**c.** 27 compared with 18 = $27 \div 18 = \frac{3}{2}$, or $1\frac{1}{2}$.

This means that $\frac{3}{2} \times 18 = 27$. (27 is three-halves or one and one-half as much as 18.)

Compare the following by the division process:

| | a. | b. |
|---|---|---|
| 1. | \$12.00 compared with \$9.00 | 5 lb. compared with 7 lb. |
| 2. | 8 balls compared with 14 balls | 2 ft. compared with 8 in. |
| 3. | 9 in. compared with 2 yd. | 32 compared with 16 |
| 4. | 8 compared with 24 | 8 compared with 3 |

Give the value of these comparisons:

| | a. | b. | c. |
|---|---|---|---|
| 5. | $10 \div 20$ | $32 \div 8$ | $25 \div 15$ |
| 6. | 7 mi. ÷ 3 mi. | 5 mi. ÷ 15 mi. | 18 apples ÷ 24 apples |

Some of the above values are in their simplest forms when the fractions are reduced to lowest terms. For example, 32 compared with 48 may be:

$$\frac{32}{48} = \frac{16}{24} = \frac{8}{12} = \frac{4}{6} = \frac{2}{3}$$

**7.** Sally and Joe like to hike. If Sally hiked for 75 min. while Joe hiked for only 50 min., how would Sally's time compare with Joe's time? How does Joe's time compare with Sally's?

**8.** How does the length of the month of April compare with the length of March? Compare using both subtraction and division.

**9.** Joe spent 35¢ for a movie, and Sally spent 75¢ for a ticket to a puppet show. How do these amounts compare using division? Give the answer two ways.

## The Ratio Idea

You have been comparing numbers by the division process on the last two pages. The comparisons that you found were really measures of relationships. This way of measuring is sometimes called finding **ratios.**

The ratio of 8 ft. compared with 12 ft. is expressed this way:

| *Division* | *Ratio* | | *1st Number* | *Ratio* | *2nd Number* |
|---|---|---|---|---|---|
| $\frac{8 \text{ ft.}}{12 \text{ ft.}}$ | $= \frac{2}{3}$ | or | 8 ft. | $= \frac{2}{3} \times$ | 12 ft. |

You can say that the ratio of 8 ft. to 12 ft. is $\frac{2}{3}$ (two-thirds). You can also say that the ratio of 8 ft. to 12 ft. is two-to-three.

**1.** What is the ratio of 12 ft. to 8 ft.?

| *Division* | *Ratio* | | *1st Number* | *Ratio* | *2nd Number* |
|---|---|---|---|---|---|
| $\frac{12 \text{ ft.}}{8 \text{ ft.}}$ | $= \frac{3}{2}$ or $1\frac{1}{2}$ | or | 12 ft. | $= \frac{3}{2} \times$ | 8 ft. |

**2.** Ratios can be whole numbers, proper fractions, or improper fractions. Tell why.

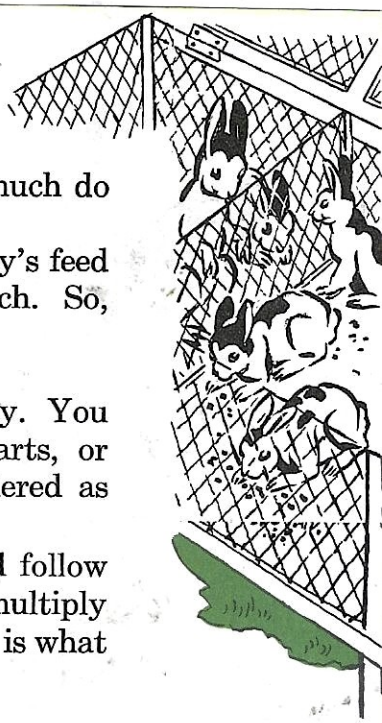
Find the numerical values of the ratios in the following:

| | **a.** | **b.** | **c.** |
|---|---|---|---|
| **3.** | 24 min. to 12 min. | 24 hr. to 18 hr. | \$9.00 to \$6.00 |
| **4.** | 12 min. to 24 min. | 18 hr. to 24 hr. | \$6.00 to \$9.00 |

**5.** Fifteen miles is to twenty miles as three is to four. Show two ways of writing this relationship using figures.

**6.** Sam had 45 min. to play after school today while Bob had only 18 min. Find the ratio of Sam's time to Bob's; of Bob's time to Sam's.

| | **a.** | | | **b.** | | |
|---|---|---|---|---|---|---|
| | *1st Number* | *Ratio* | *2nd Number* | *1st Number* | *Ratio* | *2nd Number* |
| **7.** | ? | 2 to 3 | 21 ft. | ? | $\frac{5}{4}$ | 20 in. |
| **8.** | ? | 3 to 1 | 12 | ? | $\frac{12}{3}$ | 9 |
| **9.** | 72 rd. | ? | 96 rd. | 54 lb. | ? | 81 lb. |



## Cancellation

If John's rabbits eat $\frac{3}{4}$ lb. of feed a day, how much do they eat in twelve days? $12 \times \frac{3}{4}$ lb. = ?

You know that the unit is $\frac{1}{4}$ pound, and one day's feed is 3 of them. 12 days would be 12 times as much. So,

$$12 \times \frac{3}{4} = \frac{12 \times 3}{4} = \frac{36}{4} = 9 \text{ lb.}$$ Why?

Let's look at this multiplication in a different way. You can see $\frac{3}{4}$ as 3 pounds divided into 4 equal parts, or as $\frac{1}{4}$ of 3 lb. Then your problem may be considered as 12 times $\frac{1}{4}$ of 3 lb., or $12 \times \frac{1}{4} \times 3$ lb.

You can take $\frac{1}{4}$ of 3 lb. first, if you choose, and follow with the multiplication by 12. Another way is to multiply $12 \times \frac{1}{4}$ and then multiply 3 lb. by the result. Here is what happens in this case:

$12 \times \frac{3}{4}$ lb. $= 12 \times \frac{1}{4} \times 3$ lb. = ?

$12 \times \frac{1}{4} = \frac{12}{4} = 3$. Why?

Then, $12 \times \frac{1}{4} \times 3$ lb. $= 3 \times 3$ lb. $= 9$ lb.

Now, see if this is a shorter way by which you can do this work:

$12 \times \frac{3}{4} = ?$

$\overset{3}{\cancel{12}} \times \frac{3}{\cancel{4}} = ?$

$3 \times 3 = 9$

You see from the above steps that the 12 is to be divided by 4 in the multiplication. You can do the division as shown here. This short way of dividing the 12 by 4 is called **cancellation.**

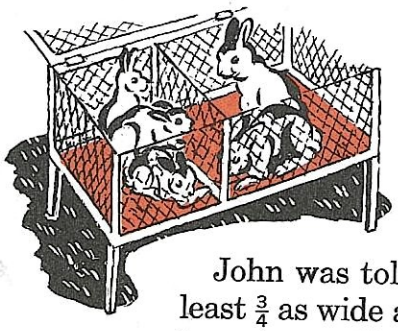
Jack's rabbits eat 9 lb. of feed in 12 days.

**1.** If Dick is able to solve $\frac{1}{3}$ of a page of problems in 1 hour, how many pages at this rate can he solve in 9 hours? $\overset{3}{\cancel{9}} \times \frac{1}{\cancel{3}} = 3 \times 1 = 3$ pages. Why?

**2.** The oil furnace in Ann's new house burns $\frac{1}{8}$ of a gallon of fuel oil an hour. How many gallons will be burned at this rate in a whole day?

**3.** Use cancellation in these exercises:

$63 \times \frac{7}{9}$ $\quad 36 \times \frac{5}{6}$ $\quad 35 \times \frac{4}{7}$ $\quad 48 \times \frac{5}{12}$ $\quad 60 \times \frac{7}{10}$



## More Cancellation

John was told that the pen for his rabbits should be at least $\frac{3}{4}$ as wide as it is long. If he has room for a pen 16 ft. long, how wide should it be?

You have seen this kind of problem before. You know that you have $\frac{3}{4}$ of 16 or $\frac{3}{4} \times 16$. The first time you met it you saw how $\frac{3}{4}$ meant $\frac{1}{4}$ taken 3 times. You, therefore, took $\frac{1}{4}$ of 16 ($4\overline{)16} = 4$) and then multiplied the result by 3, or $3 \times 4 = 12$.

You can now use cancellation in this kind of multiplication of fractions:

$\frac{3}{\cancel{4}} \times \cancel{16}^{4} = 3 \times 4 = 12$ 16 is divided by 4 by the cancellation process; the result, 4, is then multiplied by 3.

What is the difference between the cancellation method and this other method? $\frac{3}{4} \times 16 = \frac{3 \times 16}{4} = \frac{48}{4} = 12$

1. Bases on a regular hard ball diamond are 90 feet apart. If regulation softball bases are $\frac{2}{3}$ as far apart, what is the distance from home to first on a softball diamond?

2. If some younger boys have a diamond with bases $\frac{2}{3}$ as far apart as the regulation softball bases, how long are the base paths for this diamond?

Use cancellation in these exercises:

**3.** $\frac{5}{8} \times 24 = ?$
**4.** $\frac{5}{6} \times 18 = ?$
**5.** $\frac{1}{5} \times 25 = ?$
**6.** $\frac{3}{7} \times 21 = ?$
**7.** $\frac{3}{9} \times 36 = ?$
**8.** $\frac{2}{3} \times 12 = ?$

Multiply:

| | a. | b. | c. | d. | e. | f. |
|---|---|---|---|---|---|---|
| **9.** | $\frac{2}{5} \times 30$ | $\frac{5}{9} \times 45$ | $\frac{3}{4} \times 36$ | $\frac{6}{7} \times 28$ | $\frac{3}{4} \times 32$ | $\frac{3}{7} \times 14$ |
| **10.** | $\frac{5}{8} \times 24$ | $\frac{2}{5} \times 15$ | $\frac{1}{3} \times 30$ | $\frac{3}{4} \times 48$ | $\frac{7}{8} \times 16$ | $\frac{6}{5} \times 10$ |
| **11.** | $\frac{6}{7} \times 42$ | $\frac{2}{9} \times 63$ | $\frac{1}{2} \times 40$ | $\frac{5}{6} \times 60$ | $\frac{2}{9} \times 27$ | $\frac{4}{3} \times 21$ |
| **12.** | $\frac{3}{8} \times 56$ | $\frac{5}{9} \times 54$ | $\frac{2}{3} \times 60$ | $\frac{5}{4} \times 20$ | $\frac{9}{8} \times 72$ | $\frac{7}{9} \times 81$ |

## A Page of Problems

**1.** Jerry watches 11 TV shows regularly each week. If they are $\frac{1}{2}$ hour long, how much time each week does he spend at this activity?

**2.** Bill's gas model airplane was entered in the model air races. The rules allowed only $\frac{1}{12}$ of a tank of gas for the race in which he was to take part. How much gas is he allowed if a full tank holds 48 ounces?

**3.** The woman next door liked Donna's sewing so much that she offered to pay her $2.00 an hour to sew for her. If she worked $\frac{3}{4}$ of an hour, how much would she receive?

**4.** Jay rode each day on his bike just in his own driveway. He found that he traveled the $\frac{5}{8}$ of a mile from his house to the main road 12 times one day. What was his total mileage for this day?

**5.** If Pete travels on his bike at the rate of 20 mi. an hour, how far could he ride in $\frac{2}{5}$ of an hour?

**6.** How far would Pete travel in $\frac{5}{6}$ of an hour if his rate was 18 mi. per hour?

**7.** John was trying to figure how much food he bought for his rabbits for a whole year. If they eat $\frac{3}{4}$ lb. of grain each week and $\frac{2}{3}$ bu. of carrots each month, what would his figures be for each item for the year?

**8.** Sue collects stamps from all over the world. She estimates that she spends $\frac{2}{7}$ of her free time on this hobby. About how much time does she spend per day if she has about 49 hr. of free time each week?

**9.** Jack told Glenn that of his $300 collection of rare coins, $\frac{2}{15}$ of the value was in coins that were over 100 years old. What was the value of these coins?

**10.** There is approximately $\frac{11}{20}$ the amount of sunlight on the shortest day of the year as there is on the longest day. The sun shines just over 16 hr. on the longest. About how many hours does it shine on the shortest?

## Problems and Exercises

Art's father built a garage 44 ft. long. His friend, Mr. Williams, said he was going to build one about $\frac{1}{3}$ that size. How long will Mr. Williams' new garage be?

$\frac{1}{3} \times 44 = \frac{44}{3}$ or $3\overline{)44} = 14\frac{2}{3}$ ft.

$$\begin{array}{r} 14 \\ 3\overline{)44} \\ \underline{3} \\ 14 \\ \underline{12} \\ 2 \text{ ft. R} \end{array}$$

The 2 ft. remainder divided into 3 equal parts = $\frac{2}{3}$ of 1 foot = $\frac{2}{3}$ ft. Why?

Mr. Williams' new garage will be 14 ft. 8 in. long. Why?

**1.** Jimmy was given home work in three subjects. If he had 4 hours to spend altogether and wanted to spend $\frac{1}{3}$ of this time on each subject, how much time would he have for each?

**2.** The treasury of the boys' club contained $17.00. The boys planned to use $\frac{1}{4}$ of their money for a party. How much could they spend?

**3.** What is $\frac{1}{9}$ of 65?

**4.** A truck that has a gross weight of 2,110 pounds can carry $\frac{1}{2}$ of that in freight. How much freight can it carry?

**5.** Elaine's older brother, Fred, was going to be married in 2 months, and he was looking for an apartment. He was told that he should spend no more than $\frac{1}{4}$ of his monthly salary in rent. If Fred earned $325 per month, how much could he afford for rent?

| | | | | |
|---|---|---|---|---|
| **6.** | $\frac{1}{7} \times 21$ | $42 \times \frac{1}{3}$ | $\frac{1}{3} \times 72$ | $64 \times \frac{1}{8}$ |
| **7.** | $\frac{1}{12} \times 132$ | $85 \times \frac{1}{9}$ | $\frac{1}{4} \times 75$ | $96 \times \frac{1}{16}$ |
| **8.** | $\frac{1}{5} \times 85$ | $63 \times \frac{1}{7}$ | $\frac{1}{3} \times 88$ | $256 \times \frac{1}{6}$ |

**9.** Tell what it means to take $\frac{2}{7}$ of a group of things.

**10.** Explain these processes:

**a.** $\frac{2}{7} \times 14 = 2 \times \frac{1}{7} \times 14 = 2 \times \frac{1}{\cancel{7}_1} \times \frac{\cancel{14}^2}{1} = \frac{2 \times 1 \times 2}{1} = 4$

**b.** $15 \times \frac{3}{5} = \frac{\cancel{15}^3}{1} \times \frac{3}{\cancel{5}_1} = \frac{3 \times 3}{1} = 9$



## Jane's Hobby Is Cooking

Jane likes to cook so well that she always looks for new things to bake. Today she decided to make a chocolate nut cake. She planned to use only $\frac{1}{2}$ of the recipe. $\frac{1}{2}$ of the various parts will mean that she multiplies each of them by $\frac{1}{2}$.

This is the recipe that Jane intends to use:

| | |
|---|---|
| $\frac{2}{3}$ cup shortening | 1 teaspoon cinnamon |
| 2 cups sugar | $\frac{1}{2}$ teaspoon mace |
| 4 eggs | $\frac{1}{2}$ teaspoon grated nutmeg |
| 1 cup mashed potatoes | $\frac{1}{2}$ teaspoon ground cloves |
| 2 cups flour | 1 cup chopped nuts |
| $3\frac{1}{2}$ teaspoons baking powder | $\frac{2}{3}$ cup milk |

For the $\frac{2}{3}$ cup of shortening, the unit is $\frac{1}{3}$ cup. There are 2 of them, so $\frac{1}{2}$ of 2 units is $2\overline{)2} = 1$. She will use 1 of the $\frac{1}{3}$ cups. Jane could find this by cancellation:

$$\frac{1}{\cancel{2}} \times \frac{\overset{1}{\cancel{2}}}{3} = 1 \times \frac{1}{3} = \frac{1 \times 1}{3} = \frac{1}{3}$$

Jane will use $\frac{1}{3}$ cup of shortening.

When you want $\frac{1}{2}$ of a value, you divide the how-many by 2. So, in this case the division has been done by dividing the 2 in the numerator of $\frac{2}{3}$ by the 2 in the denominator of $\frac{1}{2}$. The two 2's are crossed out (cancelled) to show that the division has been completed. The quotient 1 is then multiplied by the 1 in $\frac{1}{2}$.

**1.** Multiply:

$\frac{1}{3} \times \frac{3}{5}$ $\quad$ $\frac{1}{4} \times \frac{8}{9}$ $\quad$ $\frac{1}{2} \times \frac{6}{7}$ $\quad$ $\frac{1}{5} \times \frac{10}{11}$ $\quad$ $\frac{1}{6} \times \frac{6}{7}$

## More Than One of the Equal Parts of a Fraction

Suppose Jane wanted to find $\frac{3}{4}$ of $\frac{8}{11}$. Can you help her? Does the following seem reasonable?

$\frac{3}{4} \times \frac{8}{11} = ?$

$$\frac{3}{\cancel{4}} \times \frac{\overset{2}{\cancel{8}}}{11} = \frac{3 \times 2}{11} = \frac{6}{11}$$

The $\frac{3}{4}$ means to multiply one of the 4 equal parts of $\frac{8}{11}$ by 3. One of the 4 equal parts of 8 units (size $\frac{1}{11}$) = 2. 3 such parts would be $3 \times 2 = 6$ units, or $\frac{6}{11}$.

What does the above cancellation mean?

Multiply:

**1.** $\frac{2}{3} \times \frac{6}{7}$ $\quad \frac{4}{5} \times \frac{10}{13}$ $\quad \frac{5}{6} \times \frac{12}{7}$ $\quad \frac{2}{7} \times \frac{14}{15}$ $\quad \frac{2}{9} \times \frac{18}{21}$

Another kind of exercise is the type where the number of units as given cannot be divided evenly. Tell why you can do this:

$\frac{3}{4} \times \frac{3}{5} = ?$

$\frac{3}{5} = \frac{12}{20}$ Why?

Then, $\frac{3}{4} \times \frac{3}{5} = \frac{3}{4} \times \frac{12}{20}$,

and by cancellation, you have:

$$\frac{3}{\cancel{4}} \times \frac{\overset{3}{\cancel{12}}}{20} = \frac{3 \times 3}{20} = \frac{9}{20}$$

The unit is $\frac{1}{5}$. You have 3 of them. 3 units cannot be divided into 4 equal parts evenly.

Why was the unit $\frac{1}{5}$ changed to the unit $\frac{1}{20}$? Did it change the number of units so the division could be completed?

How did you get $\frac{9}{20}$ for the product? Where was the process of changing a fraction to higher terms used?

**2.** Multiply the following by changing the number to be multiplied to higher terms:

$\frac{2}{3} \times \frac{2}{5}$ $\quad \frac{2}{7} \times \frac{2}{3}$ $\quad \frac{3}{5} \times \frac{3}{4}$ $\quad \frac{3}{2} \times \frac{5}{4}$ $\quad \frac{5}{6} \times \frac{5}{9}$

$\frac{2}{3} \times \frac{1}{2}$ gal. $\quad \frac{2}{3} \times \frac{1}{3}$ yd. $\quad \frac{2}{3} \times \frac{5}{6}$ yd. $\quad \frac{3}{4} \times \frac{7}{8}$ lb.

**3.** What would Jane have done if she wanted to use $\frac{1}{2}$ of a recipe that called for $2\frac{1}{2}$ cups of flour ($2\frac{1}{2} = \frac{5}{2}$)?

**4.** How much is $\frac{1}{2}$ of $3\frac{1}{2}$ teaspoons of baking powder?

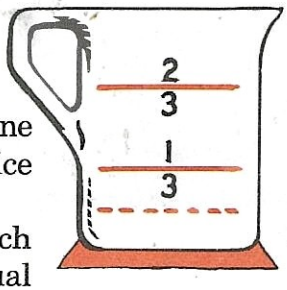
**5.** Find $\frac{2}{3}$ of $2\frac{1}{2}$ yd., $\frac{3}{4} \times 3\frac{1}{3}$ yd., $\frac{2}{5} \times 4\frac{1}{2}$ ft.

## Jane Discovers a Shortcut

Jane used a recipe one day that told her to use $\frac{1}{3}$ of a cup of molasses for flavoring. But she was using only a half recipe. She used this way to find $\frac{1}{2} \times \frac{1}{3}$:

Changing the $\frac{1}{3}$ to higher terms she had $\frac{2}{6}$:

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{\cancel{2}} \times \frac{\overset{1}{\cancel{2}}}{6} = \frac{1}{6}$$




All at once, she had an idea. It was: If $\frac{1}{3}$ of a cup is one of the 3 equal parts, then a half of $\frac{1}{3}$ cup is 1 of twice as many parts. So, she said to herself:

$$\frac{1}{2} \times \frac{1}{3} = 1 \times \frac{1}{2 \times 3} = 1 \times \frac{1}{6} = \frac{1 \times 1}{6} = \frac{1}{6}$$

The $2 \times 3$ means to divide each of the 3 parts into 2 equal smaller parts, making 6 parts, which are units of $\frac{1}{6}$ size. Then 1 of the $\frac{1}{6}$ (the how-many I am to use) makes $\frac{1}{6}$.

> To multiply two fractions, you find the product of the numerators and place the result over the product of the denominators.

**1.** Can the above method be used to multiply two fractions like $\frac{3}{4} \times \frac{8}{9}$? Let's see: $\frac{3}{4} \times \frac{8}{9} = \frac{3 \times 8}{4 \times 9} = \frac{24}{36} = ?$

Do you see that the product should be changed to lower terms? What is the answer in lowest terms?

**2.** Multiply, using the method given above:

$\frac{1}{2} \times \frac{4}{5}$ $\quad \frac{1}{2} \times \frac{8}{9}$ $\quad \frac{2}{3} \times \frac{5}{7}$ $\quad \frac{3}{4} \times \frac{3}{5}$ $\quad \frac{1}{3} \times \frac{1}{3}$

**3.** Use the rule to find $\frac{2}{3} \times 5$:

$$\frac{2}{3} \times 5 = \frac{2}{3} \times \frac{5}{1}$$
$$\frac{2}{3} \times \frac{5}{1} = \frac{2 \times 5}{3 \times 1} = \frac{10}{3} = ?$$

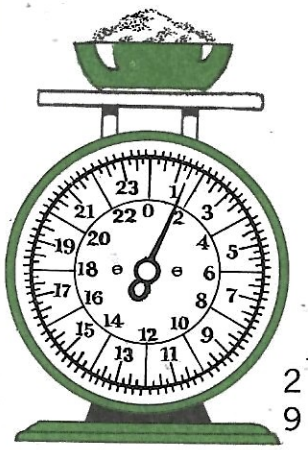
Why can you write the whole number 5 with 1 as a denominator for the size of the part?

**4.** Multiply:

$\frac{3}{4} \times 7$ $\quad \frac{2}{3} \times 4$ $\quad \frac{5}{6} \times 5$ $\quad \frac{1}{6} \times 7$ $\quad \frac{2}{5} \times 4$ $\quad \frac{2}{9} \times 5$

**5.** What does the fraction $\frac{3}{1}$ mean?

**6.** Why does the fraction $\frac{1}{1}$ equal the value 1?



## Betty Answers Her Own Question

One of Betty's hobbies is cooking. The recipe that she was using told her to use 9 oz. of an ingredient. She wanted only $\frac{2}{3}$ of the recipe.

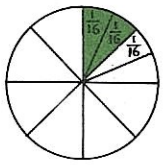
Betty already knew that she could do this, $\frac{2}{3} \times 9$, and get $2 \times 3 = 6$ oz. or $\frac{6}{16}$ lb. $= \frac{3}{8}$ lb. But, she thought to herself: 9 oz. $= \frac{9}{16}$ lb. Why can't I find $\frac{2}{3} \times \frac{9}{16}$ by dividing 9 by 3 and 16 by 2 to get $\frac{3}{8}$ lb.?

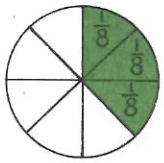
So she thought this out:

$\frac{2}{3} \times \frac{9}{16}$

$2 \times \frac{3}{16}$

$2 \times \frac{3}{16}$





One of the 3 equal parts is $\frac{3}{16}$. I need 2 of these parts. So instead of multiplying by 2, I'll make the $\frac{1}{16}$ unit twice as large by dividing 16 by 2. This is really the same as multiplying $\frac{3}{16}$ by 2. My answer is $\frac{3}{8}$. Betty called this process by the name **cross cancellation**:

$\frac{2}{3} \times \frac{3}{16} = \frac{3}{8}$

**1.** Betty found that she needed another ingredient which weighed 12 ounces. She used the multiplication $\frac{2}{3} \times \frac{12}{16}$. Find this product for her.

**2.** Find the products by cross cancellation:

$\frac{3}{4} \times \frac{16}{21} = ?$ $\frac{2}{5} \times \frac{5}{8} = ?$ $\frac{2}{3} \times \frac{21}{32} = ?$ $\frac{4}{9} \times \frac{27}{40} = ?$ $\frac{5}{6} \times \frac{12}{25} = ?$

$\frac{5}{8} \times \frac{24}{25} = ?$ $\frac{3}{8} \times \frac{4}{15} = ?$ $\frac{5}{3} \times \frac{9}{20} = ?$ $\frac{2}{3} \times \frac{6}{10} = ?$ $\frac{3}{2} \times \frac{8}{9} = ?$

**3.** A certain recipe says to use $1\frac{1}{2}$ cups of milk. If Betty wants to use only $\frac{1}{3}$ of the ingredients in the recipe, how much milk will she use?

**4.** Multiply:

$\frac{2}{3} \times 4\frac{1}{2}$ $\frac{3}{4} \times 2\frac{2}{3}$ $\frac{5}{6} \times 3\frac{3}{5}$ $\frac{2}{7} \times 10\frac{1}{2}$ $\frac{2}{5} \times 3\frac{3}{4}$

## Another Step in Cross Cancellation

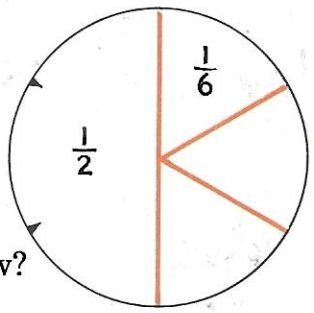
Betty found this exercise in a book:

$$\tfrac{4}{9} \times \tfrac{15}{14} = ?$$

She wondered whether she could use her old learnings to find the product.

She thought:

I know when I take $\frac{1}{3} \times \frac{1}{2}$, I must divide $\frac{1}{2}$ by 3. That's how $\frac{1}{3}$ of any number is found. I get $\frac{1}{6}$. Then the 1 in $\frac{1}{3}$ says to take 1 of these $\frac{1}{6}$ values.




How does the drawing show that $\frac{1}{2} \div 3 = \frac{1}{6}$?

Does the following tell what was done? How?

$$\tfrac{1}{3} \times \tfrac{1}{2} = 1 \times \frac{1}{3 \times 2} = 1 \times \tfrac{1}{6} = \tfrac{1}{6}$$

Betty used this same idea to multiply $\frac{4}{9} \times \frac{15}{14}$. It was like this:

$$\tfrac{4}{9} \times \tfrac{15}{14} = 4 \times \frac{15}{9 \times 14} = 4 \times \tfrac{15}{126} = \tfrac{60}{126} = \tfrac{10}{21}$$

You see that 15 units of $\frac{1}{14}$ could not be divided evenly by 9. So the unit itself $(\frac{1}{14})$ is divided. It becomes $\frac{1}{126}$. Why? Explain the other steps in the work.

Hal said, "I see a short cut. Shall I show you what it is?

$$\tfrac{4}{9} \times \tfrac{15}{14} = 4 \times \frac{15}{9 \times 14} = \frac{4 \times 15}{9 \times 14} = ?$$

"Instead of multiplying and then changing to lowest terms you can change to lowest terms before you multiply like this:

$$\frac{\overset{2}{\cancel{4}} \times \overset{5}{\cancel{15}}}{\underset{3}{\cancel{9}} \times \underset{7}{\cancel{14}}} = \frac{10}{21}$$

"I divide the 9 and 15 each by 3 and the 4 and 14 by 2. You see that this saved changing to lowest terms."

**1.** Tell what the cancellation in each of these means:

**a.** $\frac{\overset{1}{\cancel{2}}}{4} \times \frac{1}{\underset{1}{\cancel{2}}} = \frac{1 \times 1}{4 \times 1} = \frac{1}{4}$

**b.** $\overset{1}{\cancel{4}} \times \frac{3}{\underset{1}{\cancel{4}}} = \frac{3}{1} = 3$

## Practice in Cancellation

**1.** Tell why each of the following is a correct way of multiplying $\frac{5}{6} \times \frac{6}{25}$:

**a.** $\frac{5}{6} \times \frac{6}{25} = \frac{5 \times 6}{6 \times 25} = \frac{30}{150} = \frac{1}{5}$

**b.** $\frac{5}{6} \times \frac{6}{25} = \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{6}}} \times \frac{\overset{1}{\cancel{6}}}{\underset{5}{\cancel{25}}} = \frac{1}{5}$

**c.** $\frac{5}{6} \times \frac{6}{25} = 5 \times \frac{1}{\cancel{6}} \times \frac{\cancel{6}}{25} = 5 \times \frac{1}{25} = \frac{5}{25} = \frac{1}{5}$

Multiply, using cross cancellation:

| | a. | b. | c. | d. | e. |
|---|---|---|---|---|---|
| **2.** | $\frac{4}{5} \times \frac{10}{16} = ?$ | $\frac{3}{8} \times \frac{4}{9} = ?$ | $\frac{3}{4} \times \frac{4}{6} = ?$ | $\frac{1}{3} \times \frac{3}{5} = ?$ | $\frac{5}{9} \times \frac{12}{15} = ?$ |
| **3.** | $\frac{4}{5} \times \frac{10}{12} = ?$ | $\frac{5}{12} \times \frac{4}{10} = ?$ | $\frac{6}{9} \times \frac{3}{4} = ?$ | $\frac{6}{8} \times \frac{4}{12} = ?$ | $\frac{6}{7} \times \frac{35}{42} = ?$ |
| **4.** | $\frac{8}{9} \times \frac{3}{6} = ?$ | $\frac{4}{5} \times \frac{5}{12} = ?$ | $\frac{2}{3} \times \frac{3}{6} = ?$ | $\frac{6}{8} \times \frac{4}{8} = ?$ | $\frac{9}{16} \times \frac{18}{21} = ?$ |
| **5.** | $\frac{6}{7} \times \frac{21}{24} = ?$ | $\frac{3}{8} \times \frac{2}{3} = ?$ | $\frac{10}{12} \times \frac{3}{4} = ?$ | $\frac{6}{10} \times \frac{5}{8} = ?$ | $\frac{15}{32} \times \frac{24}{25} = ?$ |

**6.** When you multiply $\frac{3}{4} \times \frac{12}{15}$, why does it mean to divide 12 units into 4 equal parts and then to take 3 of them? Can you multiply the $\frac{1}{15}$ unit by 3 and get $\frac{1}{5}$? Why? Do you understand the following?

$\frac{3}{\cancel{4}} \times \frac{\overset{3}{\cancel{12}}}{15} = \cancel{3} \times \frac{3}{\cancel{15}_{5}} = \frac{3}{5}$ Explain.

**7.** Finish each of these exercises:

**a.** $\frac{3}{4} \times \frac{4}{12} = \frac{3 \times 4}{4 \times 12} = \frac{12}{48} = ?$

**b.** $\frac{3}{4} \times \frac{4}{12} = \frac{3 \times 4}{4 \times 12} = \frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{4}}}{\cancel{4} \times \cancel{12}} = ?$

**c.** $\frac{3}{4} \times \frac{4}{12} = 3 \times \frac{1}{\underset{1}{\cancel{4}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{4}{\cancel{12}}} = 3 \times \frac{1}{12} = ?$

**8.** First change the mixed numbers in these exercises to improper fractions, then multiply by using cross cancellation: $2\frac{1}{2} \times 3\frac{3}{5}$ $\quad 1\frac{1}{4} \times 2\frac{2}{5}$ $\quad 2\frac{2}{3} \times 1\frac{1}{8}$ $\quad 1\frac{1}{5} \times 2\frac{1}{12}$

**9.** John picked $3\frac{5}{8}$ bu. of head lettuce. If Sam picked $1\frac{1}{3}$ times as much as John, how many bushels did he pick?

**10.** Sue studied her arithmetic for $2\frac{1}{4}$ hours over the weekend. Betty said, "I studied $\frac{5}{3}$ times as long as Sue." How many hours did Betty study?

## Practice Problems and Exercises



**1.** Jean wanted to bake something special for a large group of girls coming to her house. She found a recipe for ladyfingers. It called for:

| | |
|---|---|
| 6 egg whites | $\frac{2}{3}$ cup bread flour |
| 4 egg yolks | $\frac{1}{4}$ teaspoon salt |
| $\frac{2}{3}$ cup powdered sugar | 1 teaspoon vanilla |

She decided she would need a double recipe. She had no trouble with the egg whites, but she had to stop to think about the powdered sugar. $2 \times \frac{2}{3}$ cup = ? cups. Jean thought that this could be done by addition:

$2 \times \frac{2}{3}$ can be $\frac{2}{3} + \frac{2}{3} = \frac{4}{3}$, or $1\frac{1}{3}$ cups. Why?

By multiplication:

$$2 \times \tfrac{2}{3} = \frac{2 \times 2}{3} = \tfrac{4}{3} = 1\tfrac{1}{3} \text{ cups}$$ Why?

Jean needed $1\frac{1}{3}$ cups of powdered sugar.

**2.** Jean wanted to triple (use 3 times) the recipe. What amount of each part must she use?

**3.** Ray was responsible for planting a 12-acre field in corn. When he had planted $\frac{5}{6}$ of the field, how many acres remained to be planted?

**4.** Alice and Fay received several orders for their sewing. If they had orders for 15 tea towels and had completed $\frac{3}{5}$ of the number, how many towels must still be done to fill the orders?

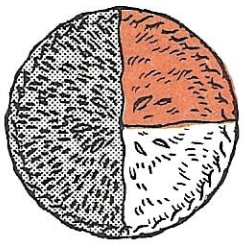


**5.** Howard was asked to shovel 3 tons of coal into the basement. When dinner time came he was $\frac{2}{3}$ finished. How many tons remained to be shoveled?

Find the products:

| | a. | b. | c. | d. | e. |
|---|---|---|---|---|---|
| **5.** | $\frac{1}{4} \times \frac{1}{3}$ | $\frac{1}{5} \times \frac{3}{8}$ | $\frac{1}{2} \times \frac{4}{5}$ | $\frac{1}{3} \times \frac{1}{3}$ | $\frac{3}{4} \times \frac{4}{5}$ |
| **6.** | $\frac{1}{4} \times 2\frac{1}{2}$ | $\frac{1}{5} \times 12$ | $\frac{2}{3} \times 9$ | $\frac{1}{4} \times \frac{8}{9}$ | $\frac{4}{7} \times \frac{14}{15}$ |





## Seeing a Rule at Work

**1.** What is $\frac{1}{2}$ of $\frac{1}{2}$ of a pie? Is this correct? $\frac{1}{2} \times \frac{1}{2} = 1 \times \frac{1}{2 \times 2}$ Why?

**2.** What has been done when you write $\frac{3}{4} \times \frac{5}{7} = 3 \times \frac{5}{28}$?

**3.** Explain: $\frac{3}{4} \times \frac{5}{7} = \frac{3 \times 5}{4 \times 7} = \frac{15}{28}$

**4.** Instead of doing cross cancellation, can you do this? $\frac{3}{4} \times \frac{8}{9} = \frac{3 \times 8}{4 \times 9} = \frac{24}{36} = \frac{2}{3}$ Explain the meaning of each step.

**5.** Why is it correct to write nine as $\frac{9}{1}$? Then, can you multiply $\frac{2}{3} \times 9$ by writing $\frac{2}{3} \times \frac{9}{1} = \frac{2 \times 9}{3 \times 1} = \frac{18}{3} = 6$? Explain. (Some people prefer to use this method.)

**6.** Have you learned a *general rule* for multiplying a fraction by a fraction? What is it?

**7.** Harry is interested in gardening. He has $\frac{1}{4}$ of an acre of land in potatoes. What part of an acre has he cultivated when he has hoed $\frac{1}{3}$ of his potatoes?

**8.** Jane's stamp book is $\frac{1}{6}$ full. If $\frac{1}{8}$ of that is American stamps, what fraction of the book contains American stamps?

**9.** Donna bought $\frac{1}{5}$ of a bolt of cotton to make aprons. She has used $\frac{1}{4}$ of it. What part of a bolt has she used?

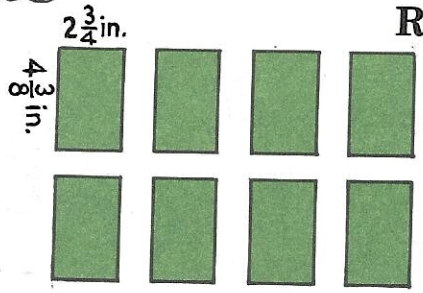
**10.** Billy took $\frac{1}{3}$ of the cookies he baked to school. He had given away approximately $\frac{1}{5}$ of what he had by recess. What fraction of his whole batch had he given away?

**11.** Ann volunteered to make $\frac{2}{5}$ of the cupcakes for the school picnic. She has completed $\frac{5}{6}$ of her job. What part of her total has she completed?

| | a. | b. | c. | d. | e. |
|---|---|---|---|---|---|
| **12.** | $\frac{2}{3} \times \frac{5}{7} = ?$ | $\frac{2}{5} \times \frac{3}{5} = ?$ | $\frac{4}{9} \times \frac{5}{7} = ?$ | $4 \times \frac{1}{4} = ?$ | $\frac{7}{12} \times \frac{5}{6} = ?$ |
| **13.** | $\frac{8}{9} \times \frac{5}{7} = ?$ | $\frac{3}{4} \times \frac{3}{5} = ?$ | $\frac{7}{8} \times \frac{3}{5} = ?$ | $\frac{1}{4} \times 12 = ?$ | $\frac{2}{3} \times 9 = ?$ |



## Rod's Hobby is Photography




Rod's camera takes pictures whose prints are $2\frac{3}{4}$ inches by $4\frac{3}{8}$ inches. He wants to mount them in an album. What size page must he have so that he can get this pattern?

He needs to do some multiplications to solve his problem. These products may be found as follows:

$$4 \times 2\tfrac{3}{4} = ?$$
$$2 \times 4\tfrac{3}{8} = ?$$

$$\begin{array}{r} 2\frac{3}{4} \text{ in.} \\ \times\ 4 \\ \hline 3 \\ 8 \\ \hline 11 \text{ in.} \end{array}$$

$$\begin{array}{r} 4\frac{3}{8} \\ \times\ 2 \\ \hline 8\frac{3}{4} \end{array}$$

Multiply the fractional part first: $4 \times \frac{3}{4} = 3$ Write 3 in ones place. Why does this give ones? $4 \times 2 = 8$ Write 8 in ones place.

Adding the products: $3 + 8 = 11$

$2 \times \frac{3}{8} = \frac{3}{4}$ Why? Write $\frac{3}{4}$ to the right of ones place. Why?

$2 \times 4 = 8$ Write 8 in ones place. The answer is $8\frac{3}{4}$ in. Rod's page in his album must be at least 11 in. by $8\frac{3}{4}$ in.

**1.** If Rod's prints are $2\frac{1}{2}$ by $3\frac{5}{8}$ inches, what size page will he need for 3 prints side by side across, and 2 up and down?

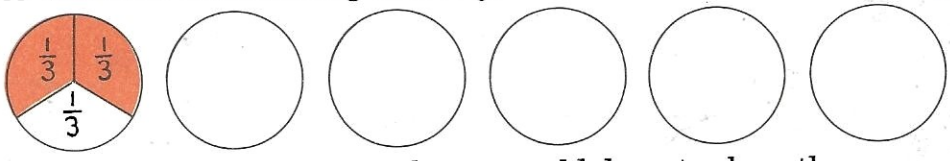
Find the products:

**2.** $2 \times 1\frac{3}{8}$ **3.** $5 \times 4\frac{1}{2}$ **4.** $4 \times 3\frac{3}{5}$ **5.** $3 \times 3\frac{3}{4}$

**6.** $2 \times 5\frac{1}{5}$ **7.** $3 \times 8\frac{5}{6}$ **8.** $5 \times 2\frac{4}{5}$ **9.** $6 \times 3\frac{2}{3}$

## Understanding Meanings—Multiplications with Fractions

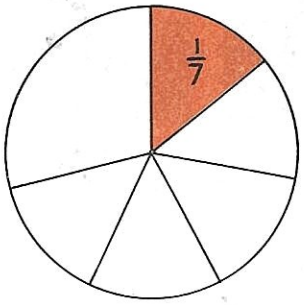
**1.** Draw a picture on a piece of paper like the following to illustrate the meaning of $6 \times \frac{2}{3}$.




**2.** Describe pictures which you would draw to show the meaning of these multiplications:

| a. | b. | c. | d. |
|---|---|---|---|
| $\frac{2}{5} \times 10 = ?$ | $\frac{3}{4} \times 12 = ?$ | $\frac{3}{7} \times 14 = ?$ | $\frac{3}{4} \times 8 = ?$ |

**3.** How could you use this picture to find $\frac{1}{2} \times \frac{5}{7}$?




**4.** Make drawings for each of these multiplications. Tell how the drawings can be checks on the actual products:

| a. | b. | c. | d. | e. |
|---|---|---|---|---|
| $\frac{1}{4} \times \frac{4}{5}$ | $\frac{3}{5} \times \frac{1}{2}$ | $\frac{2}{3} \times \frac{3}{4}$ | $\frac{3}{5} \times \frac{5}{6}$ | $\frac{3}{4} \times \frac{1}{2}$ |

**5.** Explain how $\frac{3}{4} \times \frac{3}{5}$ can equal $3 \times 3 \times \frac{1}{4} \times \frac{1}{5}$

**6.** Multiply the following:

| a. | b. | c. | d. |
|---|---|---|---|
| $\frac{3}{4} \times \frac{8}{9}$ | $\frac{2}{3} \times \frac{3}{8}$ | $\frac{6}{7} \times \frac{7}{12}$ | $\frac{4}{5} \times \frac{5}{12}$ |

**7.** Find these products in more than one way:

| a. | b. | c. | d. |
|---|---|---|---|
| $\frac{3}{8} \times \frac{4}{6}$ | $\frac{1}{3} \times \frac{3}{4}$ | $\frac{6}{8} \times \frac{4}{6}$ | $\frac{2}{3} \times \frac{9}{8}$ |

**8.** How many $\frac{1}{8}$'s are there in $\frac{3}{8}$? in $\frac{7}{8}$? in $\frac{5}{8}$?

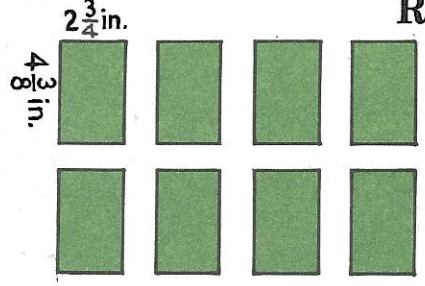
**9.** How many $\frac{1}{4}$'s are there in $\frac{1}{2}$? Tell how you found your answer.

**10.** Can you do this?

$$\frac{3}{4} \times \frac{3}{7} = \frac{3}{\cancel{4}} \times \frac{\overset{3}{\cancel{12}}}{28} = \frac{9}{28}$$ Tell why.



## Rod's Hobby is Photography




Rod's camera takes pictures whose prints are $2\frac{3}{4}$ inches by $4\frac{3}{8}$ inches. He wants to mount them in an album. What size page must he have so that he can get this pattern?

$4 \times 2\frac{3}{4} = ?$
$2 \times 4\frac{3}{8} = ?$

He needs to do some multiplications to solve his problem. These products may be found as follows:

$$\begin{array}{r} 2\frac{3}{4} \text{ in.} \\ \times \quad 4 \\ \hline 3 \\ 8 \\ \hline 11 \text{ in.} \end{array}$$

$$\begin{array}{r} 4\frac{3}{8} \\ \times \quad 2 \\ \hline 8\frac{3}{4} \end{array}$$

Multiply the fractional part first: $4 \times \frac{3}{4} = 3$ Write 3 in ones place. Why does this give ones? $4 \times 2 = 8$ Write 8 in ones place.

Adding the products: $3 + 8 = 11$

$2 \times \frac{3}{8} = \frac{3}{4}$ Why? Write $\frac{3}{4}$ to the right of ones place. Why?

$2 \times 4 = 8$ Write 8 in ones place. The answer is $8\frac{3}{4}$ in. Rod's page in his album must be at least 11 in. by $8\frac{3}{4}$ in.

**1.** If Rod's prints are $2\frac{1}{2}$ by $3\frac{5}{8}$ inches, what size page will he need for 3 prints side by side across, and 2 up and down?

Find the products:

**2.** $2 \times 1\frac{3}{8}$ **3.** $5 \times 4\frac{1}{2}$ **4.** $4 \times 3\frac{3}{5}$ **5.** $3 \times 3\frac{3}{4}$
**6.** $2 \times 5\frac{1}{5}$ **7.** $3 \times 8\frac{5}{6}$ **8.** $5 \times 2\frac{4}{5}$ **9.** $6 \times 3\frac{2}{3}$

## Rod Could Multiply a Different Way

Instead of multiplying the values in mixed numbers, Rod could change the mixed numbers to improper fractions and then find the products:

$4 \times 2\frac{3}{4} = 4 \times \frac{11}{4} = 11$ Why?

$2 \times 4\frac{3}{8} = 2 \times \frac{35}{8} = \frac{35}{4} = 8\frac{3}{4}$ Why?

1. Could these processes have been done by writing $\frac{4}{1} \times \frac{11}{4} = 11$ and $\frac{2}{1} \times \frac{35}{8} = \frac{35}{4}$? Explain.

Do these exercises, first changing the mixed numbers to improper fractions:

| | | | |
|---|---|---|---|
| **2.** $4 \times 3\frac{4}{5}$ | **3.** $6 \times 5\frac{1}{3}$ | **4.** $5 \times 4\frac{7}{8}$ | **5.** $3 \times 6\frac{3}{4}$ |
| **6.** $4 \times 4\frac{5}{6}$ | **7.** $2 \times 7\frac{3}{9}$ | **8.** $7 \times 9\frac{1}{7}$ | **9.** $5 \times 8\frac{1}{2}$ |
| **10.** $3 \times 1\frac{5}{9}$ | **11.** $4 \times 2\frac{5}{6}$ | **12.** $3 \times 4\frac{2}{3}$ | **13.** $32 \times 1\frac{9}{16}$ |

Do these exercises without changing the mixed number to a fraction:

| | a. | b. | c. | d. | e. |
|---|---|---|---|---|---|
| **14.** | $4\frac{5}{6} \times 3$ | $6\frac{7}{8} \times 4$ | $3\frac{4}{8} \times 6$ | $4\frac{2}{3} \times 5$ | $7\frac{3}{8} \times 2$ |
| **15.** | $3\frac{3}{4} \times 4$ | $4\frac{3}{5} \times 5$ | $2\frac{3}{7} \times 3$ | $6\frac{4}{9} \times 3$ | $8\frac{5}{12} \times 6$ |
| **16.** | $12\frac{5}{6} \times 8$ | $2\frac{5}{16} \times 8$ | $1\frac{1}{2} \times 12$ | $5\frac{6}{12} \times 4$ | $5\frac{5}{9} \times 6$ |
| **17.** | $12\frac{1}{3} \times 6$ | $15\frac{2}{3} \times 9$ | $10\frac{3}{4} \times 8$ | $16\frac{1}{2} \times 6$ | $30\frac{1}{4} \times 12$ |
| **18.** | $120\frac{2}{3} \times 6$ | $105\frac{2}{3} \times 10$ | $44\frac{4}{11} \times 11$ | $365\frac{1}{4} \times 4$ | $200\frac{3}{4} \times 4$ |
| **19.** | $12\frac{3}{5} \times 12$ | $10\frac{5}{9} \times 18$ | $16\frac{2}{5} \times 10$ | $24\frac{4}{9} \times 18$ | $32\frac{3}{7} \times 21$ |



## Dick Likes to Work with Wood

Dick's hobby is making things of wood. He meets many problems in fractions. For example, the dog house he is building is to be made out of a board that has to be cut into parts. He has one board 9 ft. 9 in. long, from which he wants:

**a.** $\frac{2}{3}$ of it for sides and roof.

**b.** $\frac{1}{3}$ of it for ends.

$\frac{2}{3} \times 9$ ft. 9 in. $= \frac{2}{3} \times 9\frac{3}{4}$ ft.

$\frac{1}{3} \times 9$ ft. 9 in. $= \frac{1}{3} \times 9\frac{3}{4}$ ft.

$\frac{2}{3} \times 9$ ft. 9 in. $= ?$

$\frac{1}{3} \times 9$ ft. 9 in. $= ?$

Why is it easier for Dick to find the $\frac{1}{3}$ part first?

$\frac{1}{3} \times 9\frac{3}{4} = \frac{1}{3} \times \frac{39}{4} = 3\frac{1}{4}$ ft. by the division method. Why?

$\frac{1}{\cancel{3}} \times \frac{\overset{13}{\cancel{39}}}{4} = \frac{1 \times 13}{4} = \frac{13}{4} = 3\frac{1}{4}$ ft. by the cancellation method. Why?

Then he can find the $\frac{2}{3}$ part in either of two ways:

A. $\frac{2}{3} \times 9\frac{3}{4} = \frac{\cancel{2}}{\cancel{3}} \times \frac{\overset{13}{\cancel{39}}}{\underset{2}{\cancel{4}}} = \frac{13}{2} = 6\frac{1}{2}$ ft.

B. If $\frac{1}{3}$ of $9\frac{3}{4}$ is $3\frac{1}{4}$, $\frac{2}{3}$ of $9\frac{3}{4}$ would be $2 \times 3\frac{1}{4}$, or $6\frac{1}{2}$ ft.

**1.** $\frac{1}{4} \times 2\frac{1}{2} = ?$ $\frac{3}{5} \times 3\frac{3}{4} = ?$ $\frac{5}{6} \times 3\frac{3}{5} = ?$ $\frac{3}{8} \times 5\frac{4}{5} = ?$ $\frac{2}{3} \times 8\frac{5}{8} = ?$

**2.** Try these, using mixed numbers for the measurements:

$\frac{2}{3}$ of 6 ft. 8 in. $\frac{3}{4}$ of 9 ft. 6 in. $\frac{3}{4} \times 8$ ft. 9 in. $\frac{1}{3} \times 24$ ft. 3 in.

**3.** Multiply:

| | | | |
|---|---|---|---|
| $\frac{1}{4} \times 5\frac{1}{2}$ | $\frac{3}{7} \times 4\frac{3}{7}$ | $\frac{1}{8} \times 2\frac{5}{8}$ | $\frac{2}{3} \times 4\frac{2}{9}$ |
| $\frac{1}{5} \times 1\frac{3}{5}$ | $\frac{1}{3} \times 3\frac{2}{3}$ | $\frac{5}{6} \times 3\frac{3}{4}$ | $\frac{5}{8} \times 2\frac{2}{3}$ |

## A New Kind of Multiplication

Try this multiplication:

$$\begin{array}{r} 12 \text{ ft.} \\ \times\ 2\frac{3}{4} \\ \hline 9 \\ 24 \\ \hline 33 \text{ ft.} \end{array}$$

You must multiply in parts. That is, 12 must be taken 2 times and $\frac{3}{4}$ times.

Some pupils like to multiply the fractional part first: $\frac{3}{4} \times 12 = 9$. Write 9 in ones place. Why? $2 \times 12 = 24$ Write 4 in ones place and 2 in tens place. Adding, the product is 33 ft.

Multiply:

| | | | | | |
|---|---|---|---|---|---|
| **1.** | $16 \times 3\frac{3}{4}$ | $18 \times 2\frac{2}{3}$ | $24 \times 4\frac{1}{6}$ | $25 \times 3\frac{3}{5}$ | $32 \times 5\frac{3}{8}$ |
| **2.** | $15 \times 6\frac{1}{5}$ | $56 \times 3\frac{5}{8}$ | $27 \times 2\frac{1}{3}$ | $42 \times 4\frac{5}{6}$ | $36 \times 5\frac{1}{4}$ |
| **3.** | $48 \times 9\frac{1}{8}$ | $68 \times 2\frac{3}{4}$ | $35 \times 7\frac{4}{5}$ | $54 \times 4\frac{5}{6}$ | $27 \times 6\frac{5}{9}$ |
| **4.** | $72 \times 2\frac{7}{12}$ | $96 \times 6\frac{9}{16}$ | $50 \times 3\frac{3}{10}$ | $75 \times 4\frac{6}{15}$ | $80 \times 5\frac{11}{20}$ |

**5.** George was putting boards on the side of his club house. How much wood did he need if he had to put up 18 boards, each 8 ft. 2 in. long? (use $8\frac{1}{6}$ ft.)

**6.** Since her sister was going to get married, Alice planned to make place mats for her. How much cloth would she need to make 12 mats if each were to be 1 ft. 8 in. in length? (Use $1\frac{2}{3}$ ft.)

**7.** How much cloth would Alice need in Problem 6 if she planned to make only 8 mats?

**8.** Copy and multiply:

| | | | | | |
|---|---|---|---|---|---|
| **a.** | $2\frac{2}{3} \times 33$ | $4\frac{1}{4} \times 32$ | $3\frac{4}{5} \times 30$ | $2\frac{3}{8} \times 40$ | $10\frac{2}{7} \times 14$ |
| **b.** | $5\frac{1}{2} \times 13$ | $3\frac{1}{3} \times 16$ | $4\frac{1}{4} \times 22$ | $1\frac{7}{8} \times 28$ | $3\frac{5}{6} \times 32$ |

## A Mixed Number Times a Mixed Number

If you want to multiply a mixed number by a mixed number, you can change both mixed numbers to improper fractions. Here is an example:

$$3\frac{3}{8}$$
$$\times\ 2\frac{2}{3}$$

$$2\frac{2}{3} \times 3\frac{3}{8} = \frac{8}{3} \times \frac{27}{8} \qquad \frac{\cancel{8}}{\cancel{3}} \times \frac{\overset{9}{\cancel{27}}}{\cancel{8}} = 9$$

You could write the multiplication like this:

$$\frac{8}{3} \times \frac{27}{8} = \frac{8 \times 27}{3 \times 8} = \frac{216}{24} = 9$$

$$4\frac{5}{8}$$
$$2\frac{2}{3}$$
$$3\frac{1}{12}$$
$$9\frac{1}{4}$$
$$12\frac{1}{3}$$

Of course, you do not have to change the mixed numbers into improper fractions. Study the example at the left:

You know how to find $\frac{2}{3} \times \frac{5}{8}$ and $\frac{2}{3} \times 4$. These products must be added to give the first partial product. The sum is $3\frac{1}{12}$. Check it.

Find the products, $2 \times \frac{5}{8}$ and $2 \times 4$, and add them for the second partial product. This sum is $9\frac{1}{4}$. Check it. Add the partial products. The answer is $12\frac{4}{12}$, or $12\frac{1}{3}$.

Do these as improper fractions:

| | a. | b. | c. | d. |
|---|---|---|---|---|
| **1.** | $3\frac{2}{3} \times 4\frac{3}{4}$ | $5\frac{1}{8} \times 3\frac{1}{2}$ | $\frac{5}{12} \times 6\frac{2}{3}$ | $6\frac{4}{9} \times 3\frac{5}{6}$ |
| **2.** | $4\frac{7}{12} \times 6\frac{1}{6}$ | $7\frac{5}{6} \times 3\frac{5}{16}$ | $2\frac{7}{8} \times 2\frac{7}{9}$ | $2\frac{4}{7} \times 2\frac{3}{5}$ |
| **3.** | $5\frac{3}{9} \times 11\frac{4}{7}$ | $2\frac{8}{9} \times 6\frac{2}{3}$ | $3\frac{4}{5} \times 5\frac{2}{5}$ | $6\frac{2}{8} \times 5\frac{1}{3}$ |
| **4.** | $5\frac{7}{9} \times 5\frac{5}{6}$ | $3\frac{3}{7} \times 4\frac{2}{3}$ | $5\frac{4}{7} \times 6\frac{4}{5}$ | $4\frac{1}{6} \times 7\frac{3}{5}$ |
| **5.** | $8\frac{7}{12} \times 6\frac{7}{9}$ | $7\frac{3}{4} \times 9\frac{7}{8}$ | $12\frac{5}{8} \times 5\frac{1}{3}$ | $8\frac{4}{5} \times 6\frac{2}{3}$ |
| **6.** | $5\frac{1}{2} \times 12\frac{3}{8}$ | $6\frac{2}{3} \times 7\frac{3}{8}$ | $4\frac{5}{9} \times 10\frac{6}{7}$ | $10\frac{2}{5} \times 10\frac{1}{2}$ |

**7.** Do the following multiplications two ways:

$2\frac{4}{5} \times 2\frac{1}{7}$ $\quad 3\frac{3}{4} \times 2\frac{7}{10}$ $\quad 5\frac{1}{3} \times 4\frac{3}{8}$ $\quad 4\frac{5}{7} \times 4\frac{3}{4}$

**8.** Multiply without changing to improper fractions:

$3\frac{1}{2} \times 6\frac{3}{4}$ $\quad 5\frac{2}{3} \times 7\frac{3}{5}$ $\quad 4\frac{2}{3} \times 3\frac{5}{6}$ $\quad 3\frac{3}{5} \times 5\frac{5}{6}$

$8\frac{7}{12}$ $\times\ 3\frac{3}{7}$ $\qquad$ $6\frac{3}{5}$ $\times\ 3\frac{1}{3}$ $\qquad$ $9\frac{1}{5}$ $\times\ 5\frac{1}{4}$ $\qquad$ $3\frac{5}{9}$ $\times\ 7\frac{1}{5}$ $\qquad$ $2\frac{3}{7}$ $\times\ 2\frac{2}{3}$ $\qquad$ $3\frac{6}{7}$ $\times\ 2\frac{2}{3}$



## Alice and Fay Like to Sew

Alice and Fay enjoy sewing. They make such things as towels, pot holders, and scarfs. Each Saturday morning they work on their hobby. The things that these girls make are so well done that they even take orders from neighbors. Usually they buy the materials as well as make what is wanted.

How do you think the girls would find the number of towels that could be made from 6 yd. of goods, if each towel was to be $\frac{1}{2}$ yd. long?

$6 \div \frac{1}{2} = ?$

You very probably would think: 2 towels to each yard; 6 yd. would give $6 \times 2$, or 12 towels.

When you divide by a fraction, you find how many units of a given size there are in each whole unit of the original measurement. Then you multiply this value by the number of the original whole units.

**1.** If they need $\frac{1}{4}$ yd. of goods for a pot holder, how many holders will Alice and Fay make from 6 yd.? (How many $\frac{1}{4}$ yd. in 1 yd.? in 6 yd.?)

**2.** How many dusting cloths can they make from 3 yd., if it takes $\frac{1}{3}$ yd. for each cloth?

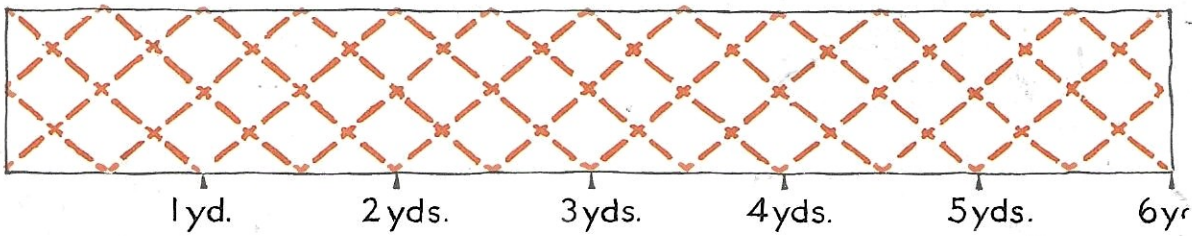
**3.** Think out the quotients in these exercises:

$4 \div \frac{1}{3}$ $\quad 5 \div \frac{1}{2}$ $\quad 3 \div \frac{1}{4}$ $\quad 2 \div \frac{1}{5}$ $\quad 6 \div \frac{1}{6}$ $\quad 2 \div \frac{1}{7}$

## How Many Towels?

How many towels, each 27 in. long, can Alice and Fay make from a piece of toweling 6 yd. long?

Some children who do not know fractions would do this problem the long way.




They would measure a length 27 in. long, time after time, counting the number of lengths, or change 6 yd. to inches and divide by 27: $27\overline{)216} = 8$ towels.

Other children who understand fractions solve this problem as you see below. They use the idea that was presented on the previous page.

27 in. $= \frac{27}{36}$ yd. $= \frac{3}{4}$ yd. How do they get this?

6 yd. $\div \frac{3}{4}$ yd. $= ?$ Why must they divide?

They first find how many $\frac{1}{4}$ yd. in 6 yd.

6 yd. $\div \frac{1}{4}$ yd. $= 24$

There are 4 of the $\frac{1}{4}$ yd. in 1 yd. So, there would be $6 \times 4 = 24$ of the $\frac{1}{4}$ yd. in 6 yd.

But the problem wants to know how many $\frac{3}{4}$ yd. in 6 yd. rather than how many $\frac{1}{4}$ yd. They think:

6 yd. $\div \frac{3}{4}$ yd. $= 8$

$\frac{3}{4}$ yd. is 3 of the units of $\frac{1}{4}$ yd. $\frac{3}{4}$ is a value 3 times as large as $\frac{1}{4}$. So there are $\frac{1}{3}$ as many $(\frac{1}{3} \times 24)$ when dividing 6 yd. by $\frac{3}{4}$ yd. as there are when dividing by $\frac{1}{4}$ yd.

**1.** Use the above method to find how many measurements of the size $\frac{2}{3}$ cup there are in 8 cups.

**2.** Divide:

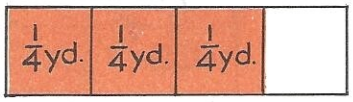
$9 \div \frac{3}{4}$ $12 \div \frac{6}{7}$ $12 \div \frac{4}{5}$ $6 \div \frac{2}{3}$ $10 \div \frac{5}{6}$ $15 \div \frac{3}{5}$

## Alice and Fay Find a Way to Divide by a Fraction

Alice and Fay discovered what they thought was a shorter way to divide by a fraction. This is the explanation that they gave to the class when dividing 6 yd. by 27 in. or $\frac{3}{4}$ yd.

"From the meaning of division by a fraction, you first find how many $\frac{3}{4}$ yd. in 1 yd., or how many towels each $\frac{3}{4}$ yd. long there are in 1 yard.

1 towel or $\frac{3}{4}$ yd. uses 3 of the $\frac{1}{4}$ yd. units.
$\frac{1}{4}$ or 1 of the $\frac{1}{4}$ yd. units is left over.




"This $\frac{1}{4}$ yd. left over is $\frac{1}{3}$ enough for another towel. Why? There is one whole $\frac{3}{4}$ yd. and enough left over for $\frac{1}{3}$ of another in 1 whole yard, or $1\frac{1}{3}$ of the $\frac{3}{4}$ yd. in 1 yd. Then, for 6 yd. you would have 6 times as many as for 1 yd., which would be $6 \times 1\frac{1}{3}$."

$6 \times 1\frac{1}{3} = 8$ towels. Why?

How do you multiply $1\frac{1}{3}$ by 6? Do you use $1\frac{1}{3}$ as a mixed number, or do you change it to an improper fraction?

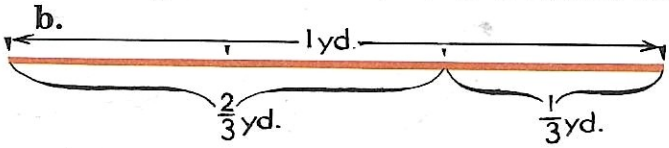
$$\begin{array}{r} 1\frac{1}{3} \\ \times\ 6 \\ \hline \end{array} \qquad \text{or} \qquad 6 \times \frac{4}{3}$$

Either will give the correct answer:

$6 \times 1\frac{1}{3} = (6 \times \frac{1}{3}) + (6 \times 1) = 8$ or $\frac{2}{\cancel{6}_1} \times \frac{\cancel{4}^2}{\cancel{3}} = 8$ Why?

**1.** How many towels $\frac{2}{3}$ yard long can be made from a piece of toweling 8 yd. long? Use two methods:

**a.** Change both measurements to inches and divide.

**b.** 

Use the method of finding how many towels can be made from 1 yard.

**2.** How many $\frac{1}{2}$ lb. boxes of candy can the girls fill from 10 lb. of hard candy?

**3.** It takes Alice and Fay $\frac{3}{4}$ hr. to make one towel. At this rate how many towels can they make in 9 hr.?

**4.** Divide:

$4 \div \frac{1}{3}$ $\quad 4 \div \frac{2}{3}$ $\quad 6 \div \frac{2}{5}$ $\quad 6 \div \frac{3}{5}$ $\quad 9 \div \frac{3}{7}$ $\quad 12 \div \frac{3}{4}$

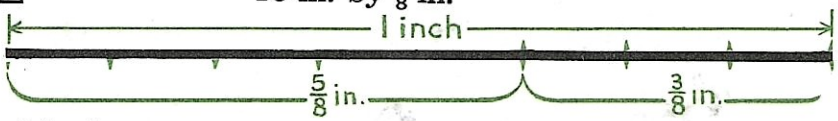


## Ted's Hobby Is Collecting Stamps

Ted wants to mount a certain kind of foreign stamp on 1 page of his album. The page is 10 in. wide, and the kind of stamp he wants to mount is $\frac{5}{8}$ in. wide. How many of this kind can be placed side by side across one page?

Ted's problem is to divide 10 in. by $\frac{5}{8}$ in.

$10 \text{ in.} \div \frac{5}{8} \text{ in.} = ?$




From 1 in. he gets one whole stamp, and he has enough room left for $\frac{3}{5}$ of another stamp. Do you see why? It takes 5 of the $\frac{1}{8}$ units for 1 stamp, and there are 3 units left toward a second stamp: 3 out of 5 is $\frac{3}{5}$. So, for each inch he can mount $1\frac{3}{5}$ stamps. Then, for 10 in., he would have 10 times as many as for 1 in., or $10 \times 1\frac{3}{5}$.

First, change the mixed number to an improper fraction: $1\frac{3}{5} = ?$

$$10 \times 1\tfrac{3}{5} = \overset{2}{\cancel{10}} \times \tfrac{8}{\cancel{5}} = 16 \text{ stamps}$$

When you wanted to divide $10 \div \frac{5}{8}$, you did it by using $10 \times \frac{8}{5}$; notice that the $\frac{5}{8}$ in the division became $\frac{8}{5}$ in the multiplication.

> When dividing a number by a fraction, invert the divisor (turn it upside down) and multiply.

**1.** Do you understand how the fraction that is the inverted divisor tells how many times the divisor is contained in 1 whole unit in the dividend? Explain.

**2.** Tell how a division by a fraction can become a multiplication by a fraction.

**3.** How many stamps $\frac{7}{16}$ in. wide can be mounted on an album page that is 14 in. wide?

*Rule—Dividing by a Fraction*

## Practice in Dividing a Fraction by a Fraction

$\frac{3}{4} \div \frac{1}{2} = ?$

You want to find how many parts or units of the size $\frac{1}{2}$ there are in $\frac{3}{4}$ of a whole unit. You know that there are $\frac{2}{1}$ of these $\frac{1}{2}$ units in 1 whole unit. So, in $\frac{3}{4}$ there are $\frac{3}{4}$ as many as there are in 1: $\frac{3}{4} \times \frac{2}{1}$.

$$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{3 \times 2}{4 \times 1} = \frac{6}{4} = 1\frac{1}{2}$$

Divide:

| | a. | b. | c. | d. | e. |
|---|---|---|---|---|---|
| 1. | $\frac{2}{3} \div \frac{3}{4}$ | $\frac{8}{15} \div \frac{5}{3}$ | $\frac{5}{9} \div \frac{5}{8}$ | $\frac{3}{4} \div \frac{7}{8}$ | $\frac{8}{15} \div \frac{4}{3}$ |
| 2. | $\frac{6}{7} \div \frac{4}{9}$ | $\frac{3}{8} \div \frac{9}{10}$ | $\frac{2}{5} \div \frac{2}{3}$ | $\frac{8}{15} \div \frac{1}{25}$ | $\frac{4}{15} \div \frac{5}{6}$ |
| 3. | $\frac{4}{5} \div \frac{2}{3}$ | $\frac{5}{6} \div \frac{8}{9}$ | $\frac{8}{9} \div \frac{2}{3}$ | $\frac{6}{11} \div \frac{6}{7}$ | $\frac{7}{3} \div \frac{7}{12}$ |
| 4. | $\frac{12}{15} \div \frac{2}{3}$ | $\frac{8}{3} \div \frac{4}{5}$ | $\frac{5}{8} \div \frac{3}{5}$ | $\frac{3}{4} \div \frac{2}{9}$ | $\frac{15}{32} \div \frac{5}{8}$ |
| 5. | $\frac{3}{7} \div \frac{3}{5}$ | $\frac{15}{32} \div \frac{5}{16}$ | $\frac{9}{25} \div \frac{3}{5}$ | $\frac{8}{21} \div \frac{4}{7}$ | $\frac{16}{5} \div \frac{8}{15}$ |
| 6. | $\frac{15}{16} \div \frac{5}{8}$ | $\frac{7}{5} \div \frac{5}{8}$ | $\frac{7}{10} \div \frac{21}{40}$ | $\frac{25}{12} \div \frac{5}{6}$ | $\frac{3}{4} \div \frac{2}{3}$ |
| 7. | $\frac{5}{9} \div \frac{10}{3}$ | $\frac{21}{36} \div \frac{7}{9}$ | $\frac{21}{20} \div \frac{7}{5}$ | $\frac{3}{8} \div \frac{5}{12}$ | $\frac{6}{7} \div \frac{7}{10}$ |
| 8. | $\frac{2}{9} \div \frac{2}{3}$ | $\frac{5}{3} \div \frac{16}{3}$ | $\frac{7}{2} \div \frac{7}{18}$ | $\frac{4}{5} \div \frac{5}{4}$ | $2\frac{2}{5} \div \frac{3}{5}$ |
| 9. | $12 \div \frac{8}{9}$ | $15 \div \frac{5}{8}$ | $9 \div \frac{3}{5}$ | $8 \div \frac{4}{5}$ | $16 \div \frac{3}{4}$ |

**10.** Divide: $\frac{2}{3}$ by $6 = ?$

This is called *dividing a fraction by a whole number.*

But you have learned that any whole number may be written as a fraction ($6 = \frac{6}{1}$).

$\frac{2}{3} \div 6 = \frac{2}{3} \div \frac{6}{1} = \frac{2}{3} \times \frac{1}{6} = \frac{1}{9}$ Why?

**11.** Divide $3\frac{3}{4}$ by 5: $\frac{15}{4} \div 5 = ?$

$$\frac{15}{4} \div 5 = \frac{15}{4} \times \frac{1}{5} = \frac{15 \times 1}{4 \times 5} = ?$$

or $$\frac{15}{4} \div 5 = \frac{\cancel{15}^{3}}{4} \times \frac{1}{\cancel{5}_{1}} = \frac{3 \times 1}{4 \times 1} = ?$$

**12.** Find the quotients:

| | | | | |
|---|---|---|---|---|
| $\frac{3}{4} \div 3$ | $\frac{6}{7} \div 2$ | $1\frac{1}{2} \div 3$ | $\frac{7}{8} \div 2$ | $\frac{9}{16} \div 3$ |
| $4\frac{1}{2} \div 6$ | $2\frac{5}{8} \div 7$ | $3\frac{5}{9} \div 8$ | $5\frac{1}{2} \div 3$ | $6\frac{2}{5} \div 16$ |
| $2\frac{1}{2} \div 1\frac{1}{2}$ | $1\frac{5}{9} \div 1\frac{2}{5}$ | $3\frac{3}{4} \div 1\frac{2}{3}$ | $\frac{27}{8} \div 1\frac{2}{7}$ | $9\frac{3}{5} \div 3\frac{1}{5}$ |

## Another Way to Divide Fractions

Another way to divide fractions does not use the rule. But the method does have meaning. Study the following:

$$\frac{3}{4} \div \frac{2}{3} = \frac{9}{12} \div \frac{8}{12} = 9 \div 8 = \frac{9}{8} = 1\frac{1}{8}$$

In this example the unlike units of the fractions (fourths and thirds) are changed to like units (twelfths) or a common denominator.

What is the common denominator shown? Is it the L.C.D.? Then, when dividing like units, all you have to do is to divide the how-many of them:

9 things ÷ 8 things of the same kind = $\frac{9}{8} = 1\frac{1}{8}$.

Divide the following:

| | | | | |
|---|---|---|---|---|
| **1.** $\frac{4}{5} \div \frac{3}{4}$ | **2.** $\frac{5}{6} \div \frac{1}{4}$ | **3.** $\frac{1}{3} \div \frac{3}{7}$ | **4.** $\frac{2}{5} \div \frac{2}{3}$ | **5.** $\frac{4}{7} \div \frac{5}{6}$ |
| **6.** $\frac{3}{8} \div \frac{7}{12}$ | **7.** $\frac{1}{6} \div \frac{8}{9}$ | **8.** $\frac{7}{12} \div \frac{3}{4}$ | **9.** $\frac{13}{22} \div \frac{8}{11}$ | **10.** $\frac{6}{7} \div \frac{9}{14}$ |
| **11.** $\frac{11}{16} \div \frac{1}{4}$ | **12.** $\frac{7}{32} \div \frac{5}{8}$ | **13.** $\frac{3}{8} \div \frac{5}{12}$ | **14.** $\frac{8}{9} \div \frac{2}{3}$ | **15.** $\frac{8}{15} \div \frac{2}{5}$ |

Do these by changing them to like units and then dividing the number of units only:

| | | | |
|---|---|---|---|
| **16.** $\frac{2}{5} \div \frac{1}{3}$ | **17.** $\frac{5}{6} \div \frac{3}{4}$ | **18.** $\frac{4}{7} \div \frac{1}{2}$ | **19.** $\frac{3}{8} \div \frac{3}{4}$ |
| **20.** $\frac{5}{6} \div \frac{2}{3}$ | **21.** $\frac{5}{12} \div \frac{3}{8}$ | **22.** $\frac{11}{16} \div \frac{1}{4}$ | **23.** $\frac{5}{9} \div \frac{2}{3}$ |
| **24.** $\frac{5}{8} \div \frac{15}{16}$ | **25.** $\frac{3}{5} \div \frac{7}{15}$ | **26.** $\frac{8}{9} \div \frac{3}{6}$ | **27.** $\frac{6}{7} \div \frac{1}{2}$ |
| **28.** $\frac{7}{8} \div \frac{3}{4}$ | **29.** $\frac{7}{9} \div \frac{5}{6}$ | **30.** $\frac{11}{16} \div \frac{7}{12}$ | **31.** $\frac{5}{9} \div \frac{3}{4}$ |

**32.** In dividing 1 by $\frac{5}{8}$, what is the answer as an improper fraction? as a mixed number?

> When the number 1 is divided by a fraction, the quotient is always the fraction inverted. This inverted fraction is called the original fraction's **reciprocal.**

| | | | |
|---|---|---|---|
| **33.** $1 \div \frac{2}{3} = ?$ | $1 \div \frac{7}{5} = ?$ | $1 \div \frac{3}{4} = ?$ | $1 \div \frac{2}{9} = ?$ |
| **34.** $1 \div 2\frac{1}{2} = ?$ | $1 \div 1\frac{1}{3} = ?$ | $1 \div 1\frac{1}{4} = ?$ | $1 \div 1\frac{7}{8} = ?$ |
| **35.** $3 \div 1\frac{1}{2} = ?$ | $5 \div 2\frac{1}{2} = ?$ | $7 \div 3\frac{1}{2} = ?$ | $9 \div 4\frac{1}{2} = ?$ |



## Handicraft Hobbies

You see here a group of children who have formed a handicraft club. They meet once a week in the basement of Alfred's home. Alfred has his own woodworking tools which the children use.

**1.** Alfred has a board 6 ft. 3 in. long. He needed several pieces 1 ft. 3 in. long. How many could be cut from the board?

Without measuring to see, he decided to use division. This was his work:

6 ft. 3 in. = 75 in.
1 ft. 3 in. = 15 in.

$$15\overline{)75}$$ 5 pieces
75

Joe said, "I can do that figuring with fractions. Watch me." This was Joe's figuring:

6 ft. 3 in. = $6\frac{1}{4}$ ft.
1 ft. 3 in. = $1\frac{1}{4}$ ft.
$6\frac{1}{4} \div 1\frac{1}{4} = \frac{25}{4} \div \frac{5}{4} = \frac{\cancel{25}^{5}}{\cancel{4}} \times \frac{\cancel{4}}{\cancel{5}} = 5$ pieces

Why was the answer labeled pieces and not feet?

**2.** Frances was making shelves for her doll's wardrobe. She was cutting pieces 1 ft. 2 in. long from a board 7 ft. 7 in. long. How many pieces could she cut from the board?

Divide:

| | a. | b. | c. | d. |
|---|---|---|---|---|
| **3.** | $4\frac{7}{8} \div 3\frac{2}{3}$ | $7\frac{4}{9} \div 4\frac{6}{7}$ | $8\frac{5}{6} \div 3\frac{2}{5}$ | $4\frac{2}{3} \div 1\frac{3}{4}$ |
| **4.** | $12\frac{2}{3} \div 10\frac{5}{8}$ | $6\frac{1}{2} \div 4\frac{3}{4}$ | $2\frac{8}{9} \div 1\frac{5}{8}$ | $9\frac{2}{5} \div 4\frac{3}{8}$ |

## Unit Review

Multiply:

| | a. | b. | c. | d. |
|---|---|---|---|---|
| 1. | $4 \times \frac{5}{8}$ | $6 \times \frac{3}{4}$ | $5 \times \frac{4}{7}$ | $9 \times \frac{3}{5}$ |
| 2. | $\frac{3}{4} \times 16$ | $\frac{5}{8} \times 12$ | $\frac{4}{7} \times 21$ | $\frac{3}{5} \times 35$ |
| 3. | $\frac{3}{4} \times \frac{16}{27}$ | $\frac{4}{5} \times \frac{25}{28}$ | $\frac{3}{7} \times \frac{14}{27}$ | $\frac{8}{9} \times \frac{27}{64}$ |
| 4. | $3 \times 2\frac{1}{2}$ | $5 \times 4\frac{1}{7}$ | $12 \times 6\frac{2}{3}$ | $9 \times 5\frac{3}{8}$ |
| 5. | $\frac{1}{3} \times 13\frac{1}{2}$ | $\frac{1}{5} \times 11\frac{3}{4}$ | $\frac{3}{4} \times 16\frac{1}{3}$ | $\frac{5}{6} \times 24\frac{1}{2}$ |
| 6. | $3\frac{1}{3} \times \frac{1}{4}$ | $4\frac{1}{8} \times \frac{2}{5}$ | $6\frac{3}{4} \times \frac{5}{8}$ | $5\frac{5}{7} \times \frac{2}{3}$ |
| 7. | $4\frac{1}{4} \times 2\frac{1}{8}$ | $3\frac{3}{5} \times 5\frac{1}{6}$ | $2\frac{5}{7} \times 4\frac{3}{8}$ | $9\frac{4}{5} \times 6\frac{1}{7}$ |

Divide:

| | | | | |
|---|---|---|---|---|
| 8. | $3 \div \frac{3}{5}$ | $7 \div \frac{4}{5}$ | $5 \div \frac{7}{8}$ | $8 \div \frac{5}{6}$ |
| 9. | $\frac{4}{5} \div \frac{8}{15}$ | $\frac{3}{8} \div \frac{3}{4}$ | $\frac{4}{7} \div \frac{16}{21}$ | $\frac{5}{9} \div \frac{1}{3}$ |
| 10. | $2\frac{3}{4} \div \frac{2}{3}$ | $3\frac{1}{6} \div \frac{5}{6}$ | $4\frac{5}{8} \div \frac{3}{4}$ | $6\frac{5}{7} \div \frac{14}{15}$ |
| 11. | $\frac{5}{8} \div 2\frac{1}{2}$ | $\frac{2}{3} \div 4\frac{8}{9}$ | $\frac{9}{4} \div 6\frac{3}{4}$ | $\frac{2}{15} \div 1\frac{2}{3}$ |
| 12. | $3\frac{3}{5} \div 2\frac{7}{8}$ | $4\frac{1}{3} \div 5\frac{5}{6}$ | $6\frac{7}{8} \div 5\frac{4}{7}$ | $7\frac{1}{4} \div 11\frac{2}{3}$ |

**13.** Elaine had 4 pails of feed to last her horses for 5 days. What part of a pail can she give them so they will be fed an equal amount each day?

**14.** If John gives his rabbits 2 heads of lettuce for each 3-day period, how much does he give them each day (equal parts, $\frac{1}{3} \times 2 = ?$)?

**15.** Divide:

| | | | |
|---|---|---|---|
| $\frac{3}{4} \div \frac{1}{4}$ | $\frac{5}{8} \div \frac{10}{11}$ | $3\frac{2}{3} \div 1\frac{5}{6}$ | $10 \div \frac{5}{6}$ |
| $3\frac{1}{2} \div \frac{3}{4}$ | $15\frac{1}{3} \div 2\frac{5}{6}$ | $\frac{2}{3} \div \frac{2}{3}$ | $\frac{6}{9} \div \frac{1}{3}$ |
| $\frac{1}{2} \div 6$ | $\frac{2}{3} \div 8$ | $\frac{3}{4} \div 12$ | $1\frac{3}{5} \div 10$ |
| $8 \div \frac{1}{4}$ | $10 \div \frac{2}{5}$ | $15 \div 1\frac{2}{3}$ | $14 \div 1\frac{2}{5}$ |

**16.** Alice shared 4 cakes equally with 4 friends. How much did each child receive?

**17.** John saved 6 dollars. If he had the whole amount changed to half-dollars, how many would he have?

**18.** Fay practices on the piano $\frac{3}{4}$ hr. daily. How many hours does she practice in 4 weeks?

## Unit Test

**1.** Jim wanted \$12.00 to buy a pair of skates. His father offered to pay the balance if he would earn $\frac{3}{4}$ of the amount. How much must Jim earn to get the skates?

**2.** Evan feeds his rabbits $\frac{2}{3}$ of a pound of feed per day. How many pounds does he feed them in 30 days?

**3.** Holly rode 8 miles on her bike in 1 hour. At that rate, how far could she ride in $\frac{1}{3}$ of an hour?

**4.** Sue wanted a pony but didn't know where she could keep it. Her uncle was willing to let her use $\frac{1}{5}$ of his barn space. How much space could she have if the barn was 40 feet long?

**5.** Ann is making $\frac{2}{3}$ of a cookie recipe. How much sugar should she use if the recipe calls for $\frac{3}{4}$ of a cup?

**6.** Guy was planting potatoes on $\frac{1}{3}$ of an acre. What part of an acre had he planted when he was $\frac{1}{2}$ finished?

**7.** Russ had 9 oz. of candy. If he offered to give $\frac{1}{3}$ to Ann, what fraction of a pound would he have left?

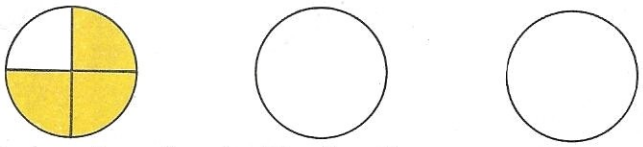
**8.** If it takes $\frac{2}{3}$ yd. of cloth to make a towel, how many towels can be made from 8 yd. of cloth?

**9.** How many kerchiefs $1\frac{1}{4}$ yd. long can be made from a piece of cloth $8\frac{1}{3}$ yd. long?

**10.** $\frac{3}{6} \div \frac{1}{2}$ **11.** $\frac{4}{5} \div \frac{2}{3}$ **12.** $\frac{11}{12} \div \frac{8}{9}$ **13.** $3\frac{1}{6} \div \frac{2}{3}$

**14.** $4\frac{5}{8} \div \frac{3}{4}$ **15.** $15\frac{4}{9} \div \frac{1}{3}$ **16.** $3\frac{7}{8} \div 2\frac{2}{5}$ **17.** $6\frac{4}{7} \div 5\frac{7}{8}$

**18.** Prove by using these pictures that you have a right to invert the fraction and multiply in $3 \div \frac{3}{4}$.



Divide by changing to like fractions:

| | a. | b. | c. | d. |
|---|---|---|---|---|
| **19.** | $\frac{5}{6} \div \frac{3}{4} = ?$ | $\frac{7}{8} \div \frac{2}{3} = ?$ | $\frac{2}{15} \div \frac{2}{13} = ?$ | $\frac{6}{1} \div \frac{2}{3} = ?$ |
| **20.** | $\frac{5}{8} \div \frac{3}{2} = ?$ | $2\frac{1}{2} \div \frac{5}{4} = ?$ | $\frac{7}{8} \div 1\frac{3}{4} = ?$ | $\frac{8}{9} \div 1\frac{3}{5} = ?$ |

# UNIT 8

# Using One Number System



Numbering grows from measuring; if there were no measurement, there would be no numbers.

Have you ever thought about the measures you use? There are: **a.** the hour divided into 60 minutes; **b.** the foot divided into 12 inches; **c.** the pound divided into 16 ounces; **d.** the gallon divided into 4 quarts; and so on. If you write the values of the parts in fractional form, they are all different. They are different because there is no common system that is used to divide all basic units into the same number of parts.

You have found that these different kinds of fractional parts sometimes make arithmetic more difficult. Do you suppose that there is anything that could be done about it? Do you know of any countries in the world that have a simpler way of dealing with the fractional parts of wholes?

All common measures could have their basic units divided into 10, 100, or 1,000 parts. You have seen this used many ways. For example, the speedometer on your father's car gives miles divided into 10 equal parts; the thermometer that the nurse or doctor uses can give degrees divided into 10 equal parts, and so on.

When number values are related by 10's, either as wholes or parts, we have what are called **decimals**. The weatherman, the surveyor, and the timekeeper may all use decimals. Let's study some of the uses of decimals in our own environment.

**1.** Read this value: $12.75. Tell how you know that the places belong in a tens system.

**2.** What is the standard unit in our money system?

**3.** How many dimes in a dollar?

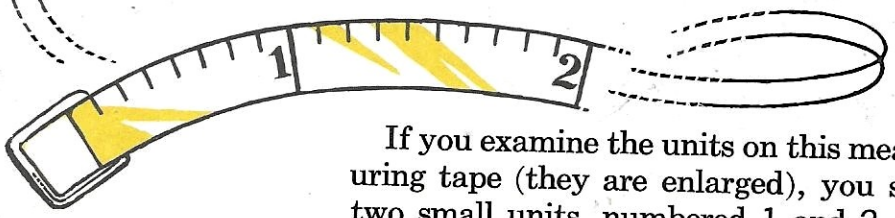
**4.** The dime is what part of 1 dollar?

**5.** How many tenths of a mile will the bus in the picture have gone when the speedometer reads 7 mi.?



## Using Decimals in Measuring

Any number tells how much or how many. Sometimes you measure something and record its value in different units. When units are so related that 10 of one kind makes 1 of another kind, you are using a decimal relation.




If you examine the units on this measuring tape (they are enlarged), you see two small units, numbered 1 and 2, as parts of a foot. They are 2 of the 10 equal parts into which the foot has been divided. Then these small units are divided into 10 equal parts. Thus the foot has 100 equal parts. Why?

You might have a measurement that would come out even, like 326 feet. This number, or any whole number, uses the decimal relation. Why? 326 = 3 hundreds, 2 tens, and 6 ones.

10 ones = 1 ten; 10 tens = 1 hundred.

Why does the number $124\frac{6}{10}$ show the decimal relation? Do you see numbering by tens in such a number as $64\frac{27}{100}$? If you remember that there is an understood addition in mixed numbers, it is clear that $64\frac{27}{100} = 64 + \frac{27}{100}$ or, in other parts, $64 + \frac{27}{100} = 64 + \frac{20}{100} + \frac{7}{100} = 64 + \frac{2}{10} + \frac{7}{100}$, and you have number relations in 10's.

10 hundredths $(\frac{10}{100})$ = 1 tenth $(\frac{1}{10})$; 10 tenths $(\frac{10}{10})$ = 1.

Every value of the number system can be written in the tens relation.

**1.** Tell how money values belong in a tens-relation system.

**2.** How could you change $6\frac{1}{2}$ to the tens-relation system?

**3.** Why is it impossible to find the exact value of $3\frac{1}{3}$ in the tens system?

# Studying Whole Numbers as Decimals

## 40 Million Hunting Licenses Sold

This headline appeared in a newspaper. It tells in round numbers how many licenses were sold. Can you read the number 40,000,000 and know that it says 40 million? Tell how.

The numbering system is made up of tens. Do you see how each place value is 10 times as large as the one on its right? Since the tens relation is always present as you go from one place value to the next, you say that *whole numbers are decimals.*

| 10 hundred-millions = 1 billion | 10 ten-millions = 1 hundred million | 10 millions = 1 ten-million | 10 hundred-thousands = 1 million | 10 ten-thousands = 1 hundred-thousand | 10 thousands = 1 ten-thousand | 10 hundreds = 1 thousand | 10 tens = 1 hundred | 10 ones = 1 ten | Ones |
|---|---|---|---|---|---|---|---|---|---|
| 1 , | 4 | 8 | 5 , | 1 | 2 | 9 , | 6 | 7 | 3 |

**1.** How many tens in 480? in 5,280?

**2.** How many hundreds in 5,280? in 43,250?

**3.** What figure in the above box is in thousand-thousands place?

**4.** How many thousands in 43,250?

Can you read the number in the box? It is one billion, four hundred eighty-five million, one hundred twenty-nine thousand, six hundred seventy-three.

**5.** Read these numbers: 4,568; 31,763; 120,063; 465,101; 3,730,210.

**6.** Write these numbers: ten thousand, three hundred seventy-five; one hundred twenty thousand; one million, four hundred sixty-three thousand, one hundred twenty.

## Writing Fractional Parts in Tenths

There have been times when you wanted to divide a whole into 10 equal parts. If you used some of these parts, you called them tenths.

**1.** Read each of these fractions:

$\frac{1}{10}$ $\frac{5}{10}$ $\frac{3}{10}$ $\frac{9}{10}$ $\frac{7}{10}$ $\frac{6}{10}$ $\frac{8}{10}$ $\frac{1}{10}$ $\frac{4}{10}$ $\frac{13}{10}$

**2.** Write each of these fractions:

| | | | |
|---|---|---|---|
| nine tenths | six tenths | eight tenths | four tenths |
| seven tenths | five tenths | one tenth | three tenths |
| ten tenths | two tenths | eleven tenths | twelve tenths |

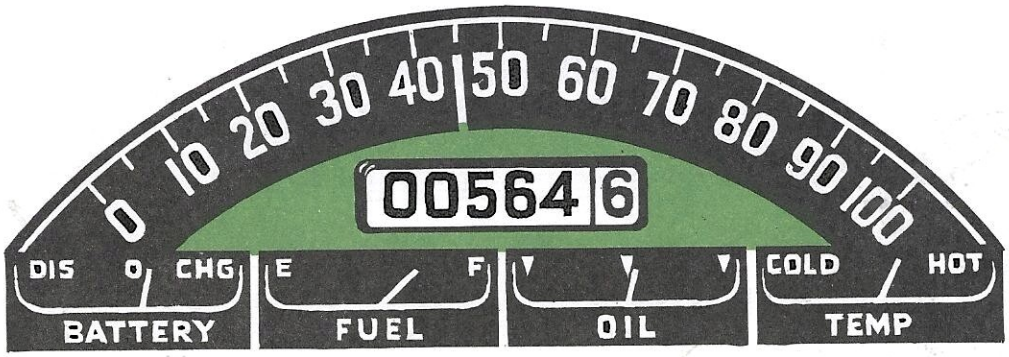
**3.** Ann was given $\frac{3}{10}$ of the family garden. If she were to put $\frac{1}{10}$ of her share into corn, how much would she have left for other things?

**4.** John, Paul, and Bill collected and sold empty bottles. This was one of their ways of earning money. Once, John owned $\frac{4}{10}$ of all the bottles they had collected. He needed $\frac{1}{10}$ of all the bottles to buy a car for his electric train. How many cars could he buy, using all of his share of the bottles?

**5.** A cake was divided into 10 equal parts. If Sally and Alice each ate $\frac{1}{10}$ of the cake, Mary ate $\frac{2}{10}$, and Kate ate $\frac{4}{10}$, how much did the 4 girls eat in all? How much of the cake was left?

**6.** Frank was able to work only $\frac{7}{10}$ of an hour after school each day during the past week. How much time in all did he work in the 5 days?

| | | |
|---|---|---|
| **7.** $\frac{3}{10} + \frac{6}{10} = ?$ | **8.** $\frac{2}{10} + \frac{5}{10} = ?$ | **9.** $\frac{8}{10} + \frac{1}{10} = ?$ |
| **10.** $\frac{4}{10} + \frac{5}{10} = ?$ | **11.** $\frac{7}{10} - \frac{4}{10} = ?$ | **12.** $\frac{6}{10} - \frac{2}{10} = ?$ |
| **13.** $\frac{9}{10} - \frac{7}{10} = ?$ | **14.** $\frac{5}{10} - \frac{4}{10} = ?$ | **15.** $\frac{6}{10} - \frac{2}{10} = ?$ |
| **16.** $\frac{4}{10} \times \frac{3}{10} = ?$ | **17.** $\frac{7}{10} \times \frac{4}{10} = ?$ | **18.** $\frac{3}{10} \div \frac{5}{10} = ?$ |
| **19.** $\frac{9}{10} \div \frac{3}{10} = ?$ | **20.** $\frac{7}{10} \div \frac{5}{10} = ?$ | **21.** $\frac{4}{10} \div \frac{6}{10} = ?$ |




## Riding in an Automobile

These are the standard instruments on the dashboard of an automobile. You recognize the largest as being the speedometer. It shows how many miles per hour the car is traveling. Also, there is a little rectangular space with numbers which shows the total number of miles that the car has traveled. This instrument that tells the miles traveled uses the tens relation, or decimal system.

You have all ridden in an automobile. Do you know how to tell distances on the instrument panel? What is the reading on this page? Do you know that the last number is tenths of 1 mile? You could write this as $564\frac{6}{10}$ mi.

Sometimes the colors in the panel tell you that the units are different. Perhaps you may have noticed that the tenths wheel travels at a different speed from the others. Does it travel faster or slower? Why?

Write these values using common fractions for the parts of a whole unit:

**1.** The earth is traveling through space at approximately 19 and 2 tenths miles per second.

**2.** A good driver can get as much as 26 and 7 tenths miles on a gallon of gas from the average car.

**3.** At one time the average family consisted of 4 and 3 tenths members.

**4.** There are approximately 30 and 4 tenths days in the average month.




## The Decimal Notation

Suppose that this is the instrument on an automobile that shows the number of miles traveled. The reading is five thousand, two hundred seventy-six and six tenths miles. You can write this as $5{,}276\frac{6}{10}$ mi.

The number system, when you are numbering by tens, gives you a new way of writing these fractional parts. You know that in a number like 5,276, it takes 10 ones to make a ten, 10 tens to make a hundred, and so on. Or, going in the opposite direction, a one is $\frac{1}{10}$ of a ten, and a ten is $\frac{1}{10}$ of a hundred.

Why is 7 tens $\frac{7}{10}$ of a hundred?

Now, if there was a number on the right of ones, it would be tenths of ones. You do not need to write its denominator of 10. Its position tells its value—tenths. $5{,}276\frac{6}{10}$ can be written without a denominator 10. Put a period after the ones, like this: 5,276.6, to tell where the whole ones end and the fractional parts in the tens numbering begin. The period is called the **decimal point.**

Tell why you can write $65\frac{4}{10}$ as 65.4 ft.

This way of writing the *numbering by tens* is called the *decimal notation.*

Because of the fact that you can make use of decimals for all numbers, they are said to belong in the *decimal system.*

Even though $16\frac{3}{10}$ uses the tens relation it is not called a decimal. It is still called a mixed number. If it is written 16.3, then it is called a decimal.

You have to be careful to identify the decimal point in a value when it comes at the end of a sentence. For example, read the number 456.7. Which period in this number is the decimal point that tells place values?

## The Kinds of Decimals

There are three kinds of numbers in the decimal system:

**a.** All whole numbers use numbering by tens, so they are called **whole number decimals** (427 is a whole number decimal).

**b.** When you have mixed numbers, where the fractional parts are expressed in the decimal system, they are called **mixed decimals** (32.6 is a mixed decimal).

**c.** If a value is a part of a whole unit and is expressed in the decimal system, this is called a **decimal fraction** (0.2 is a decimal fraction).

Remember: When numbers use the common fraction form, they are not called decimals. A number must be written in the decimal notation to be called a decimal.

**1.** A zero (0) is often put in ones place when the value is a decimal fraction, like 0.2. Can you give a reason for doing this?

**2.** Before decimals were studied, $23\frac{8}{10}$ was called a mixed number. Now, you can write its value as 23.8. This can be called a mixed decimal. Why?

**3.** The number $48\frac{3}{4}$ is a mixed number. Why, in its present form, is it not a decimal? a mixed decimal?

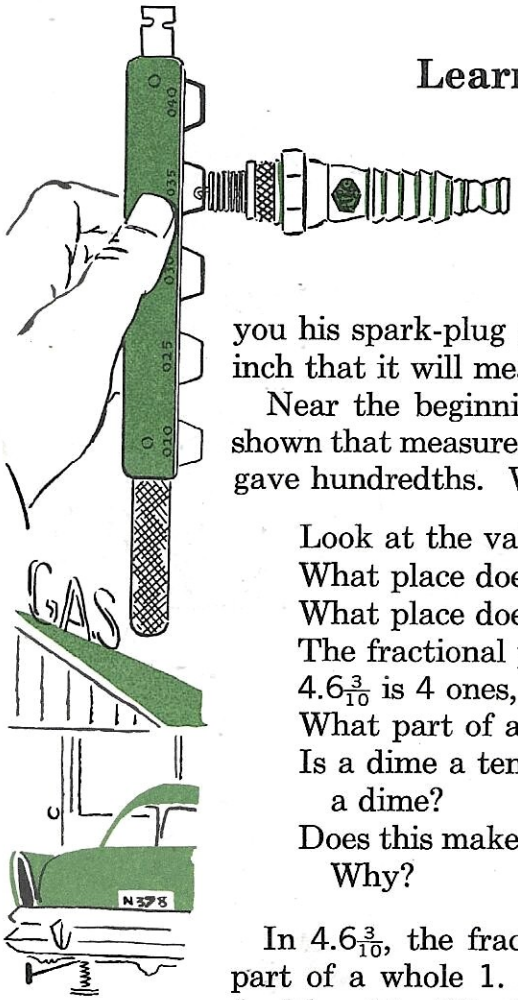
**4.** There are countries that use the decimal system in all of their measures. Find the names of some of these. Find what country was the first to adopt this system.

**5.** Which of these values would be decimals and which would be mixed numbers, if money were written this way?
\$.7 $\$1\frac{2}{10}$ \$0.3 $\$125\frac{4}{10}$ \$4,258.6 \$12,675.5 $\$\frac{5}{10}$
How would you write them as money values in the usual way? Tell why.

**6.** Why is \$0.5 equal to 5 dimes? one-half dollar?

**7.** Is \$0.8 equal to 80 cents? Why or why not?

**8.** If $\frac{7}{10} + \frac{5}{10} = \frac{12}{10}$, then why is the sum of 0.7 and 0.5 = 1.2?




## Learning About Hundredth Parts

Have you ever seen a garage mechanic's gauge for measuring the openings on the spark plugs of a car? Stop at a garage soon and ask the attendant to show you his spark-plug gauge, and to tell you the parts of an inch that it will measure.

Near the beginning of the unit a measuring tape was shown that measured units in tenths of tenth parts. These gave hundredths. Why?

Look at the value $4.6\frac{3}{10}$ inches.
What place does the 4 hold?
What place does the 6 hold?
The fractional part $\frac{3}{10}$ is a part of what unit?
$4.6\frac{3}{10}$ is 4 ones, 6 tenths, and $\frac{3}{10}$ of a tenth. Why?
What part of a whole one is a tenth of a tenth?
Is a dime a tenth of a dollar and a penny a tenth of a dime?
Does this make a penny a hundredth part of a dollar? Why?

In $4.6\frac{3}{10}$, the fractional part of a tenth is a fractional part of a whole 1. Thus, 3 tenths of a tenth is 3 hundredths of 1. Why?

$4.6\frac{3}{10}$ is written as 4.63 in the decimal notation.

The place held by 3 is called **hundredths**; it is hundredths of a whole 1.

4.63 is read as 4 and 63 hundredths, rather than 4 and 6 tenths and 3 hundredths.

4.63 = 4 ones + 6 tenths + 3 hundredths. 6 tenths = 60 hundredths. Why?

Then, 4 ones + 60 hundredths + 3 hundredths gives 4 ones and 63 hundredths.

4.63 ft. = $4\frac{63}{100}$ ft. Why?




## The Most Common Use of the Tens Relations

The amount of money \$89.75 is an example of decimal notation. What place is held by each of the figures?

You read this as 89 dollars and 75 cents. (Did you know that the word *cent* means hundredth part?)

The place of the 5 cents is a hundredth part of what place?

The place of the 7 is a tenth of what place?

**1.** If you read the 75 as cents, what does it mean?

**2.** What use is made of the decimal point in writing money values?

**3.** Why can \$89.75 be written as $\$89\frac{75}{100}$?

**4.** What coin represents the place of 7 in this value? Why?

**5.** Write each of these another way:

\$375.83 \$29.32 \$465.23 \$39.66 \$3765.44

$\$27\frac{25}{100}$ $\$436\frac{82}{100}$ $\$146\frac{18}{100}$ $\$391\frac{65}{100}$ $\$2302\frac{87}{100}$

**6.** In writing sixteen dollars and seventy-five cents, where does the decimal point go? Why?

**7.** In \$16.25, what is the place value of the 2? Why? What is the place value of the 5? Why? In coins, what would the 2 be? the 5?

**8.** In Problem 7 the amount after the decimal point is read twenty-five cents. Why? Does cents explain the place values? How?

**9.** Does $\$16.25 = \$16\frac{25}{100}$? Why?

**10.** Write as decimals:

**a.** Jill has two dollars, six dimes, and a nickel.

**b.** Ann earned four dollars and three quarters.

**c.** Bill bought a football that cost six dollars, three quarters, two dimes, and four pennies.

## Adding Decimals

You add both mixed decimals and decimal fractions by the same method used when working with whole numbers. Like units are placed in the same columns, that is, hundredths are placed under hundredths, tenths under tenths, and so on. Does this make the decimal points in the different numbers fall into a column? Why?

When a sum in one column is above 9, you have values that must be added to the next higher place. Why?

$$\begin{array}{r} 56.4 \\ +\ 32.9 \\ \hline 89.3 \end{array}$$

1. How many tenths does it take to make a whole unit?

2. Mary walked 0.3 of a mile from her house to the store, 0.2 of a mile from the store to Polly's house, and 0.4 of a mile back home. How far did she walk in all?

Add:

| | | | | |
|---|---|---|---|---|
| 3. \$2.75 | \$13.27 | \$82.61 | \$37.48 | \$3.46 |
| 1.26 | 3.53 | 18.89 | 64.73 | 4.21 |
| | | | | 3.65 |
| 4. \$16.31 | \$625.52 | \$37.84 | 2.8 | 30.6 |
| 3.65 | 9.36 | 463.29 | 14.6 | 42.56 |
| 22.47 | 28.80 | 19.55 | 9.7 | 8.09 |
| 5. \$36.21 | \$46.78 | \$47.74 | \$19.21 | 8.32 |
| 42.75 | 92.12 | 90.05 | 3.48 | 14.6 |
| 86.29 | 75.86 | 36.67 | 46.83 | 7.28 |
| 56.48 | 37.22 | 41.22 | 147.92 | 15.09 |
| 27.91 | 61.53 | 68.14 | 5.50 | 7.77 |

6. How many hundredths are equal to a tenth?

7. Tom earned \$4.27 mowing lawns and \$6.73 weeding a neighbor's garden. How much did he earn altogether?

8. Center City had 3 rainy days in 1 week in April. The weather bureau reported that the rainfall for these different days was 1.21 in., 0.35 in., and 1.08 in. How much rain fell in Center City during the week?

## Subtracting Decimals

Subtracting decimals is like subtracting any number. You should remember that all whole numbers are called decimals because the tens relation exists in them just as it does in mixed decimals and decimal fractions.

**1.** When you subtract hundreds and the figure in tens place in the top number is smaller than the number in tens place in the bottom number, what do you do?

**2.** Subtract:

$$\begin{array}{r} 42.4 \\ -\ 19.5 \\ \hline 22.9 \end{array}$$

Why must you use 1 of the ones to give you more of the tenths?

From how many tenths has the 5 tenths been subtracted?



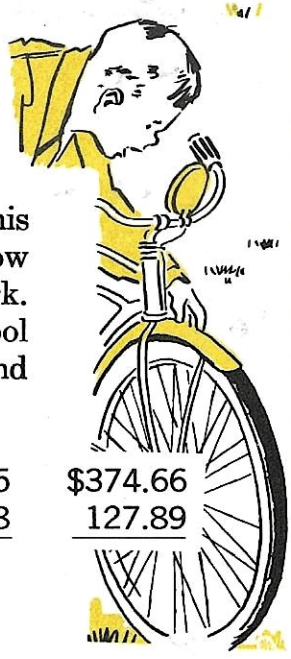
**3.** Pete ran the one-hundred-yard dash in 14.2 seconds. Some college men can run it in 9.6 seconds. How much faster is the college men's speed?

$$\begin{array}{r} 14.2 \\ -\ 9.6 \\ \hline \end{array}$$

You first subtract: 12 tenths – 6 tenths. Why? Then: 13 – 9. Why?

**4.** Jack has an instrument on his bicycle that measures the miles he travels. If it reads 26.3 before he left for a ride and 31.8 after he returned, how far did he ride?

$$\begin{array}{r} 31.8 \\ -\ 26.3 \\ \hline \end{array}$$



**5.** Jack found that he could use the speedometer on his bicycle for a number of things. He wanted to know whether his school ground was larger than the city park. The reading was 47.6 before riding around the school ground and 51.2 after. It was 53.5 before riding around the park and 57.4 after. Which is larger?

Subtract:

| **6.** | | | | | | |
|---|---|---|---|---|---|---|
| \$65.54 | \$326.48 | \$896.95 | \$900.00 | 62.5 | \$374.66 |
| 9.37 | 154.63 | 543.58 | 46.82 | 37.8 | 127.89 |

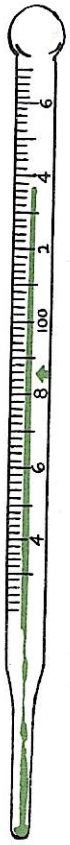


## Practice Problems and Exercises

Here is a thermometer like the one that the doctor is using. How is it marked? Read it as whole degrees and parts of a degree. Would you say that the person's temperature, as a mixed number, is $103\frac{4}{5}$? Explain.

**1.** Can you tell from the thermometer what a person's normal temperature is? How far is this temperature above normal?

**2.** What would Eileen's temperature be if the red line came up to the second mark after the 4? How do you know? Can you write this as a mixed number? How would you write it as a decimal?



**3.** Write the temperature reading of $98\frac{3}{5}$ as a decimal.

**4.** How much above normal would a person's temperature be if the thermometer showed a value that could be read as 100.2?

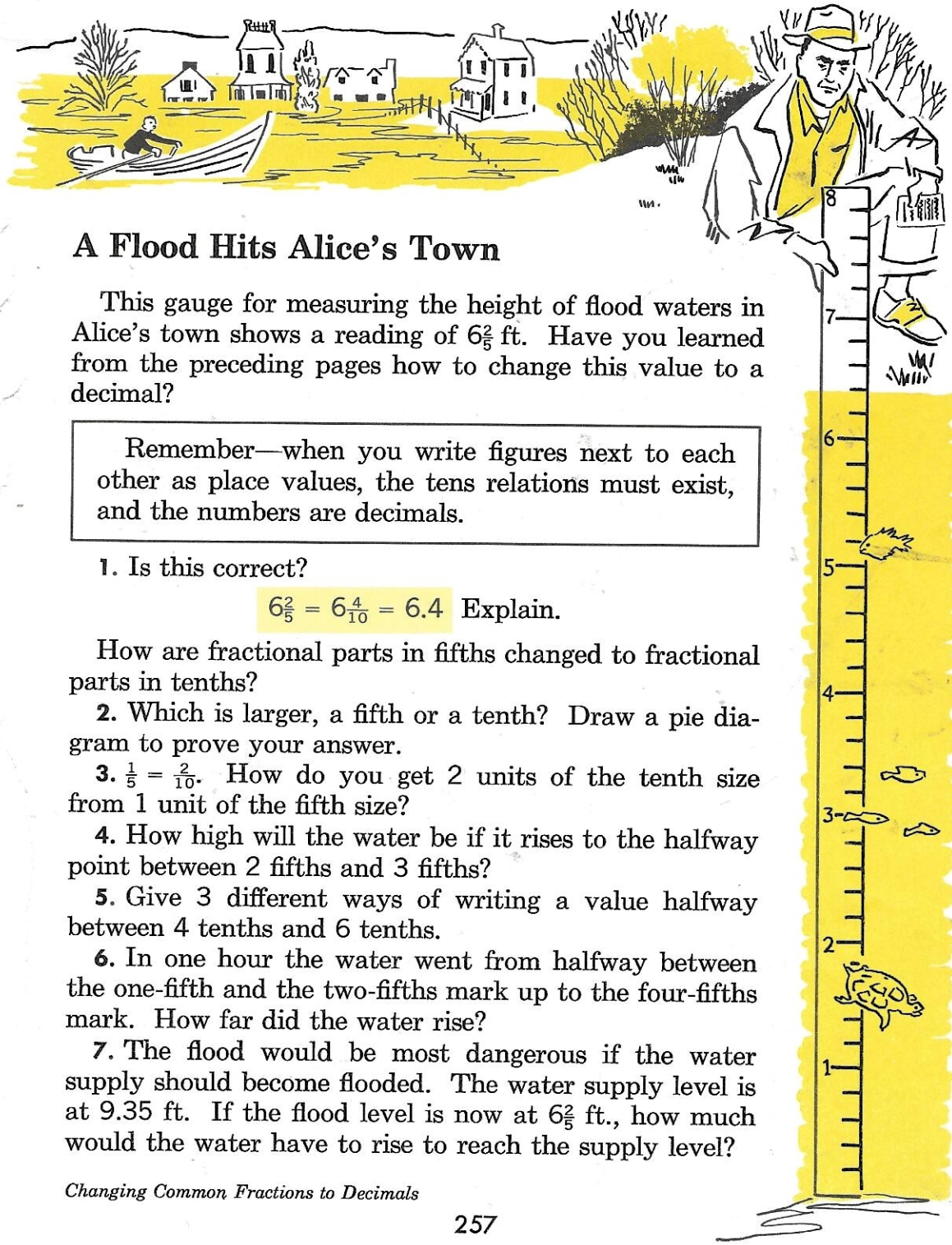
**5.** Add these values as mixed numbers, then write the sums as decimals:

| a. | b. | c. | d. |
|---|---|---|---|
| $27\frac{5}{10}$ | $46\frac{7}{10}$ | $126\frac{3}{10}$ | $632\frac{4}{10}$ |
| $32\frac{4}{10}$ | $39\frac{2}{10}$ | $145\frac{19}{100}$ | $538\frac{27}{100}$ |

**6.** Subtract as mixed numbers:

| a. | b. | c. | d. |
|---|---|---|---|
| $26\frac{6}{10}$ | $41\frac{8}{10}$ | $76\frac{92}{100}$ | $47\frac{4}{10}$ |
| $13\frac{5}{10}$ | $36\frac{3}{10}$ | $32\frac{4}{10}$ | $26\frac{28}{100}$ |

**7.** Change the exercises in No. 6 to decimals, then subtract and check your answers with those that were obtained when you used mixed numbers.




## A Flood Hits Alice's Town

This gauge for measuring the height of flood waters in Alice's town shows a reading of $6\frac{2}{5}$ ft. Have you learned from the preceding pages how to change this value to a decimal?

> Remember—when you write figures next to each other as place values, the tens relations must exist, and the numbers are decimals.

**1.** Is this correct?

$$6\tfrac{2}{5} = 6\tfrac{4}{10} = 6.4$$ Explain.

How are fractional parts in fifths changed to fractional parts in tenths?

**2.** Which is larger, a fifth or a tenth? Draw a pie diagram to prove your answer.

**3.** $\frac{1}{5} = \frac{2}{10}$. How do you get 2 units of the tenth size from 1 unit of the fifth size?

**4.** How high will the water be if it rises to the halfway point between 2 fifths and 3 fifths?

**5.** Give 3 different ways of writing a value halfway between 4 tenths and 6 tenths.

**6.** In one hour the water went from halfway between the one-fifth and the two-fifths mark up to the four-fifths mark. How far did the water rise?

**7.** The flood would be most dangerous if the water supply should become flooded. The water supply level is at 9.35 ft. If the flood level is now at $6\frac{2}{5}$ ft., how much would the water have to rise to reach the supply level?

## How Well Have You Learned?

**1.** What is the basic unit of our numbering system? Why is it correct to say that our numbering system is a decimal system?

**2.** Are all numbers that have decimal points decimals? Can a number be a decimal if it does not have a decimal point?

**3.** What is a decimal fraction?

**4.** What does the decimal point mean in a decimal fraction?

**5.** Can money be written as whole decimals? fractions? decimal fractions? Illustrate each way.

**6.** Why aren't the fractional parts of a dollar called decimal fractions? What term is used as a substitute for decimals in referring to this kind of money?

**7.** Describe in your own words what 0.01 means.

**8.** Tell what .01 is in terms of tenths relations.

**9.** What is the difference between a decimal fraction and a common fraction? Which is easier to use? Is this true all of the time?

**10.** Which is larger, 6 and 30 hundredths or 6 and 3 tenths? Why?

**11.** Which is larger, $5\frac{1}{2}$ or 5 and 50 hundredths?

**12.** Read the following decimals:

| | | | | | |
|---|---|---|---|---|---|
| 4378 | 6957.0 | 34.6 | 8976.00 | 8.98 | 37.65 |
| 11306.80 | 9.46 | 0.46 | 3546.23 | 0.05 | 0.40 |

**13.** How many tenths of tens are there in 42.5?

**14.** How many hundredths of tens are there in 42.5?

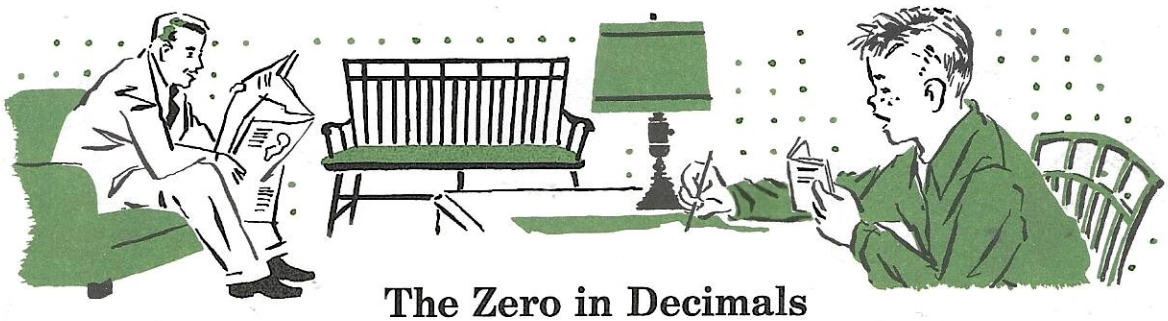
**15.** How many tenths could there be in 5.24?

**16.** How many tenths of tenths are there in 365.24?

**17.** Write the following as common fractions:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0.2 | 0.8 | 0.3 | 0.7 | 0.9 | 0.6 | 1.0 | 0.32 |
| 0.21 | 4.6 | .88 | .65 | 60 | 1.00 | 0.02 | |

**18.** Is it possible to write \$6.75 as $\$6\frac{75}{100}$? as 67 dimes + 5 cents? as 675 cents?



## The Zero in Decimals

Jimmy earned enough money last summer to open a savings account at a bank. His balance was \$6.40. He now has \$3.05 to add to this.

How many dollars does Jimmy now have in all? How many dimes could he have? pennies?

Jimmy asked his father why the zero was needed in \$6.40.

His father said, "Our money is always measured in dollars and cents. Since a cent is $\frac{1}{100}$ of a dollar, figures in money must always show how many hundredths there are."

Jimmy said that he could see that, but he couldn't see why they would need the zero in a figure like 6.40 miles. He was told that the zero on the end shows that the measurement is more exact, that the value is correct to the nearest hundredth of 1.

His father asked him if he could leave out the zero in \$3.05. Of course, Jimmy could understand why not. There are not any tenths in this value and, if you leave the zero out, the 5 hundredths could be called 5 tenths.

The *zero* is used to indicate *not-any* or *none* in the place where it is found, just as it is in whole numbers. Notice how zero is used here:

8.05 is read as eight and five hundredths.

6.30 is read as six and thirty hundredths.

70.08 is read as seventy and eight hundredths.

1. Read these decimals:

7.5 18.09 17.20 61.03 1376.18 4792.30 937.06



## Buying Coal by the Ton

The Brown family lives on a farm. Their coal supply was getting low, so one day Mr. Brown and his son, Pete, went to town to buy a load.

When the load was weighed, the dealer said, "You have 3,500 pounds, or $1\frac{3}{4}$ tons of coal."

Was the dealer correct when he said that 3,500 lb. = $1\frac{3}{4}$ tons? Let us figure it:

2000 lb. = 1 ton (t.)

3500 lb. = 1 t. + 1500 lb. Why?

$$3500 \text{ lb.} = 1 \text{ t.} + \frac{1500}{2000} \text{ t.} \quad \text{Why?}$$

3500 lb. = $1\frac{3}{4}$ t.

Pete wanted to change this mixed number to a mixed decimal. First, he thought like this:

If $1\frac{3}{4}$ were dollars, I know that the decimal value would be \$1.75, because 75¢ or \$.75 is $\frac{3}{4}$ of a dollar.

So, I know that $1\frac{3}{4} = 1.75$.

Then, he examined $\$1\frac{2}{5}$: $\$1\frac{2}{5} = \$1.$ and ?.

$$1\tfrac{2}{5} = 1\tfrac{40}{100} = \$1.40$$

Why must Pete write hundredths?

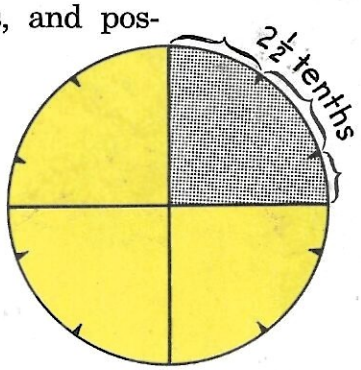
Write in decimal form, showing hundredths:

**1.** $\$\frac{1}{4}$ $\$\frac{1}{5}$ $\$\frac{7}{10}$ $\$\frac{15}{10}$ $\$\frac{65}{100}$ $\$1\frac{1}{2}$ $\$\frac{5}{2}$ $\$\frac{7}{4}$

**2.** $\$\frac{1}{2} = \$\frac{?}{100} = \$0.?$ $\$\frac{2}{5} = \$\frac{?}{100} = \$0.?$ $\$\frac{1}{4} = \$\frac{?}{100} = \$0.?$

*Common Fractions in Money Changed to Hundredths in Decimals (Cents)*

Pete knew, of course, that $1\frac{3}{4}$ t. was not dollars. To change the $\frac{3}{4}$ from its fractional form to decimals, he must think in terms of the tens relationship. He knew that decimal fractions contain tenths, hundredths, and possibly other places.




He first tried to change the $\frac{3}{4}$ to tenths:

$1\frac{3}{4} = 1\frac{?}{10}$

$1\frac{3}{4} = 1\frac{7\frac{1}{2}}{10}$

Tenths follow ones. In changing fourths to tenths, there would have to be $2\frac{1}{2}$ of the tenths in each fourth ($4 \times 2\frac{1}{2}$ tenths = 10 tenths or the whole).

Since there would be $2\frac{1}{2}$ of these for each fourth, in 3 fourths you would have $3 \times 2\frac{1}{2}$, or $7\frac{1}{2}$ of the tenths.

Pete then thought of a better way. Since the number of tenths did not come out even, perhaps the next smaller part in the tens system or hundredths would come out even:

$1\frac{3}{4} = 1\frac{?}{100}$

$1\frac{3}{4} = 1\frac{75}{100}$

In changing fourths to hundredths, each is divided into 25 equal parts. Why? Then, there would be 25 times as many of the hundredths as of the fourths, or $25 \times 3 = 75$.

The $1\frac{75}{100}$ can be written as a decimal: $1\frac{75}{100} = 1.75$ Why?

Mr. Brown and Pete had a load of coal weighing $1\frac{3}{4}$ t. or 1.75 t.

Change the following to hundredths in decimals:

**1.** $\frac{3}{5} = \frac{?}{100} = ?$ $\frac{1}{4} = \frac{?}{100} = ?$ $\frac{4}{10} = \frac{?}{100} = ?$ $\frac{1}{2} = \frac{?}{100} = ?$ $\frac{5}{4} = \frac{?}{100} = ?$

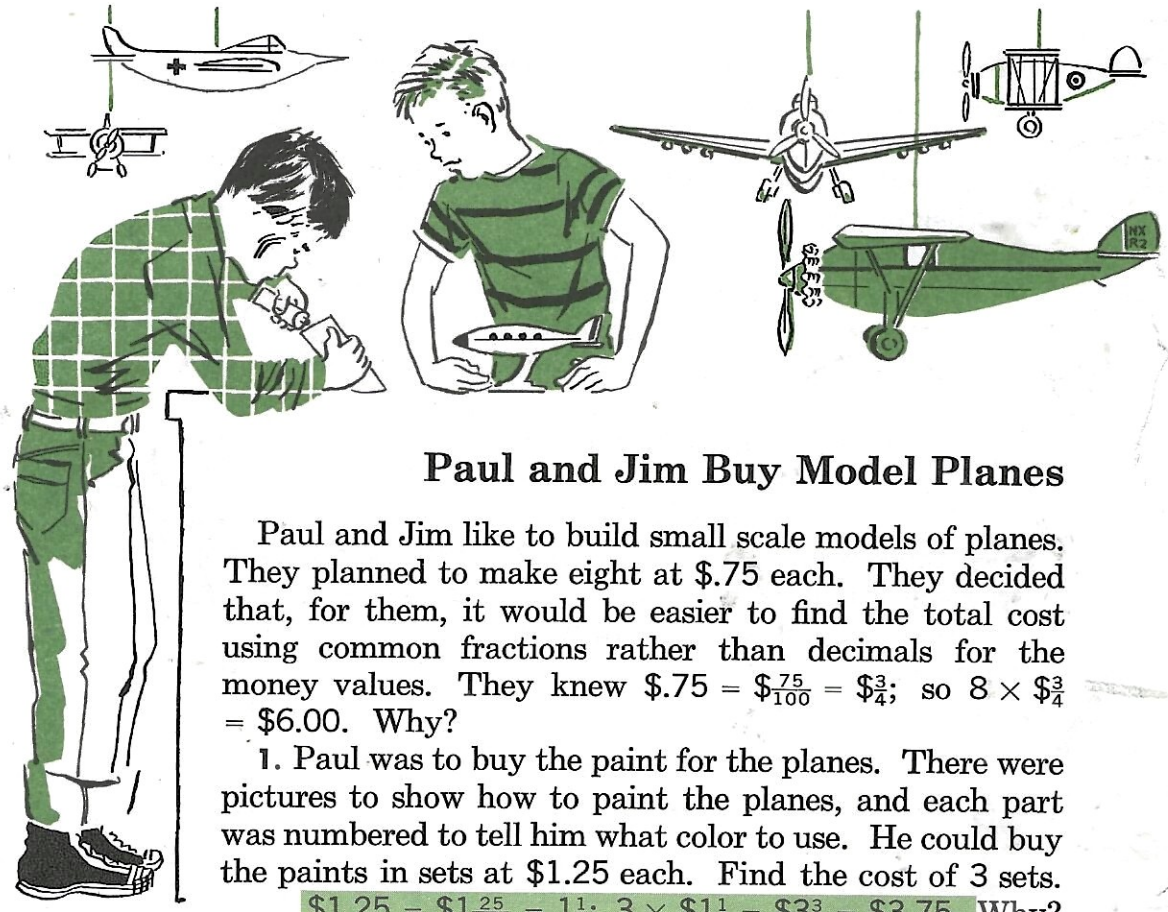
**2.** $\frac{1}{5} = \frac{?}{100} = 0.\ ?$ $\frac{2}{5} = \frac{?}{100} = 0.\ ?$ $\frac{4}{5} = \frac{?}{100} = 0.\ ?$

**3.** $\frac{1}{10} = \frac{?}{100} = 0.\ ?$ $\frac{3}{10} = \frac{?}{100} = 0.\ ?$ $\frac{7}{10} = \frac{?}{100} = 0.\ ?$

**4.** $\frac{1}{20} = \frac{?}{100} = 0.\ ?$ $\frac{1}{50} = \frac{?}{100} = 0.\ ?$ $\frac{6}{5} = \frac{?}{100} = 0.\ ?$

**5.** $\frac{4}{5} = 0.\ ?$ $\frac{5}{2} = 2.\ ?$ $\frac{3}{2} = 1.\ ?$ $\frac{7}{4} = ?$

**6.** Change these mixed numbers to decimal form in hundredths: $1\frac{2}{5}$ $1\frac{7}{10}$ $2\frac{1}{4}$ $12\frac{1}{2}$ $5\frac{3}{4}$ $1\frac{3}{10}$ $1\frac{3}{20}$ $2\frac{7}{50}$



## Paul and Jim Buy Model Planes

Paul and Jim like to build small scale models of planes. They planned to make eight at \$.75 each. They decided that, for them, it would be easier to find the total cost using common fractions rather than decimals for the money values. They knew $\$.75 = \$\frac{75}{100} = \$\frac{3}{4}$; so $8 \times \$\frac{3}{4} = \$6.00$. Why?

**1.** Paul was to buy the paint for the planes. There were pictures to show how to paint the planes, and each part was numbered to tell him what color to use. He could buy the paints in sets at \$1.25 each. Find the cost of 3 sets.

$\$1.25 = \$1\frac{25}{100} = 1\frac{1}{4}$; $3 \times \$1\frac{1}{4} = \$3\frac{3}{4} = \$3.75$ Why?

**2.** Change the fractional parts of these decimals to common fractions in lowest terms:

| **a.** | **b.** | **c.** | **d.** |
|---|---|---|---|
| $0.42 = \frac{42}{100} = ?$ | $0.84 = ?$ | $1.76 = \frac{176}{100} = ?$ | $0.04 = ?$ |
| $0.36 = ? = ?$ | $0.22 = ?$ | $55.60 = ?$ | $0.25 = ?$ |

**3.** Multiply 1.75 by 4, first changing the decimal to a mixed number.

**4.** Find the product of 1.25 and 1.5 after changing them first to mixed numbers and then to improper fractions.

**5.** Subtract as decimals, then check your answers by using values as mixed numbers or improper fractions:

| | | | | | |
|---|---|---|---|---|---|
| 4.25 | 3.64 | 5.50 | 16.60 | 9.36 | 8.75 |
| 2.75 | 2.40 | 2.80 | 4.75 | 4.20 | 6.25 |

## Check Your Understanding

Here is what you should know about changing common fractions to decimal fractions:

**a.** First, try to change the parts that are given in the common fraction to tenths.

**b.** If the number of parts of the tenth size gives you a whole number, your decimal fraction will be tenths.

**c.** If you do not have a whole number of tenths, then use the next smaller part in the tens numbering system, which is hundredths. When you have a whole number of hundredths, you can write the decimal fraction.

**1.** Since $1\frac{3}{4} = 1\frac{7\frac{1}{2}}{10}$, can you write it as $1.7\frac{1}{2}$? Explain.

**2.** Can you do this?

$$1\tfrac{3}{4} = 1\frac{7\frac{1}{2}}{10} = 1.7\tfrac{1}{2} = 1.7\tfrac{5}{10} = 1.75.$$ Explain.

**3.** Write $\frac{1}{4}$ in common fraction form, expressed in tenths.

**4.** Show how to change fourths to hundredths in one step, using $5\frac{1}{4} = 5\frac{?}{100}$. What is the mixed decimal?

**5.** Change to common fractions in hundredths:

$\frac{9}{20}$ $\frac{7}{10}$ $\frac{4}{5}$ $\frac{16}{25}$ $\frac{13}{50}$ $\frac{5}{4}$ $\frac{8}{10}$ $\frac{3}{20}$

**6.** Change these to decimal fractions in hundredths:

$\frac{3}{50} = \frac{}{100} = ?$ $\frac{7}{25} = \frac{}{100} = ?$ $\frac{3}{10} = \frac{}{100} = ?$

$\frac{19}{25} = \frac{}{100} = ?$ $\frac{11}{20} = \frac{}{100} = ?$ $\frac{7}{4} = \frac{}{100} = ?$

**7.** If you divide this line into 4 equal parts and then into 100 equal parts:

---

**a.** What will be the size of the unit when there are 4 equal parts?

**b.** What will 1 part or unit be called when there are 100 equal parts?

**c.** How many parts of the hundredth size will there be in each part of the fourth size?

## Jim and His Father Examine Their Tax Bill

Tax Statement — Valuation $3100

After March 1st taxes must be paid to the County Treasurer.

With 4% penalty and $\frac{1}{2}$ of 1% interest per month until paid.

School District 1

| Tax Items | Mills | Amount |
|---|---|---|
| Co. (General) | 1,276 | 39.56 |
| Sch. (General) | 2,005 | 62.16 |
| Sch. Debt | 1,845 | 57.20 |
| Total | | 158.92 |
| Collection Fee | | 1.59 |

In examining their tax bill Jim and his father saw that they were paying taxes on three separate items, each at a different rate. Jim's father explained that the amount of their taxes was based upon the valuation of their property and the rate.

*The amount of tax is found by multiplying the valuation by the rate.*

When Jim asked what rate meant, his father said, "There are so many **mills** or tenths of a cent charged for each dollar of valuation."

"In the case of this bill the county (general) taxes are $1.276 on each $100.00." Jim said, "I could read the dollar and twenty-seven cents, but what is the 6?"

The six, his father explained, is $\frac{6}{10}$ of a cent or 6 mills.

Now, what is the name of the place held by the 6 in 1.276? You can figure it out like this:

**a.** 7 is in hundredths place, and the place of the 6 is tenths of hundredths. What do you get when you take a tenth of a hundredth (divide a hundredth part into 10 equal parts)?

**b.** The third place after the whole ones can be named by thinking the value you get by taking tenths of tenths of tenths of 1.

Both **a** and **b** give you a thousandth part, so the place is named thousandths.

## Changing Common Fractions to Thousandths in Decimals

How can Tom change 2,750 lb. to tons, expressed as a decimal?

First, as a fraction, 2,750 lb. equals $1\frac{3}{8}$ tons. Why?

$1\frac{3}{8} = 1\frac{?}{10}$, not appropriate = uneven tenths. Why?
$1\frac{3}{8} = 1\frac{?}{100}$, not appropriate = uneven hundredths. Why?
$1\frac{3}{8} = 1\frac{?}{1000}$, appropriate = each eighth subdivided 125 times. Why? So 3 of them give 375 of this part.

Is it correct to write $1\frac{375}{1000}$ as 1.375? Why? You can understand why by writing $1\frac{3}{8} = 1\frac{3\frac{3}{4}}{10} = 1.3\frac{3}{4} = 1.3\frac{7\frac{1}{2}}{10}$ $= 1.37\frac{1}{2} = 1.37\frac{5}{10} = 1.375$

**1.** Elaine bought all the candy she could for the change she had in her purse. She bought $2\frac{5}{8}$ pounds. How would this be written as a decimal?

**2.** If a test had 40 questions and you had 27 right, what is the decimal that tells 27 out of 40 parts?

**3.** Hank bought $2\frac{7}{8}$ lb. of steak for his dinner. What is the decimal equivalent of this weight?

**4.** Write $2\frac{3}{4}$ as a mixed decimal which is correct in thousandths place.

**5.** Change $\frac{9}{20}$ as a common fraction to hundredths; to thousandths; to a decimal fraction in hundredths; to a decimal fraction in thousandths.

**6.** How many mills in:

**a.** \$1.175 **b.** \$2.025 **c.** \$0.375 **d.** \$.25 **e.** 3.5¢ **f.** 10¢

**7.** Change $7\frac{3}{8}$ to a mixed decimal in thousandths, and name all the place values.

**8.** Write the decimal value for these:

**a.** $0.05\frac{1}{10}$ **b.** $0.03\frac{2}{5}$ **c.** $0.10\frac{7}{10}$ **d.** $1.0\frac{1}{4}$ **e.** $0.00\frac{1}{5}$

**9.** Explain why these are correct:

**a.** $\frac{1275}{1000} = 1.275$ **b.** $\frac{624}{1000} = 0.624$ **c.** $\frac{5}{1000} = 0.005$

## Place Values in Decimals

| billions | 100 millions | 10 millions | millions | 100 thousands | 10 thousands | thousands | hundreds | tens | ones | tenths | hundredths | thousandths | 10 thousandths | 100 thousandths | millionths |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

Can you explain this table? Is it the story of the tens numbering system?

**1.** A ten is 10 times ? .

**2.** A hundred is 10 times ? , or 100 times ? .

**3.** A thousand is 10 times ? , or 1000 times ? .

**4.** A tenth is $\frac{1}{10}$ of ? .

**5.** A hundredth is $\frac{1}{10}$ of ? , or $\frac{1}{100}$ of ? .

**6.** A thousandth is $\frac{1}{10}$ of ? , or $\frac{1}{1000}$ of ? .

**7.** Why is a hundredth $\frac{1}{10}$ of $\frac{1}{10}$ of 1?

**8.** Why is a hundredth 10 times as great as a thousandth?

**9.** Examine the table to find what place is a tenth of thousandths.

**10.** What place is 100 times as large as thousandths? as hundredths? as tenths? as ones?

**11.** Naming the parts on the right side of the table as fractional units, what unit is a hundredth as large as a thousandth?

**12.** Write the number one thousand and one thousandth.

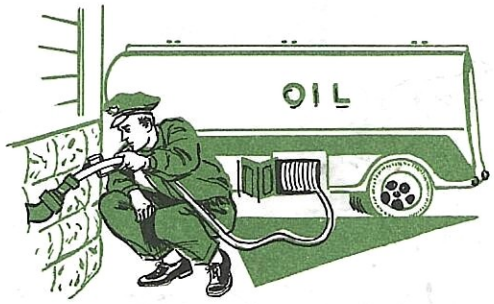
**13.** Read these values:

| a. | b. | c. | d. | e. |
|---|---|---|---|---|
| 44.03 | 4.045 | 7.003 | 4.210 | 0.037 |
| 27.605 | 2.105 | 0.025 | 3.100 | 0.005 |

**14.** Find the sums of the items in Exercise 13.

**15.** Subtract the values given for each item in Exercise 13.

## Estimating Fuel Oil Needs




Mr. Day has a new oil burner to heat his home. He was told that the average amount of oil used in cold weather, for a house the size of his, was 274.5 gal. a month. He wanted to know what he could expect to use in all, during the 4 winter months of December, January, February, and March.

$$\begin{array}{r} 274.5 \\ \times\ 4 \\ \hline \end{array}$$

Mrs. Brown, who has a smaller house, uses less oil per month. Her average consumption in cold weather last year was 128.34 gal. a month. She, too, wanted to know how much oil she could expect to use during 4 winter months.

$$\begin{array}{r} 128.34 \\ \times\ 4 \\ \hline \end{array}$$

Problems like these can be solved by multiplication, if you know how to handle decimals. Study this general method:

$$\begin{array}{r} 65.8 \\ \times\ 12 \\ \hline \end{array}$$

When you multiply by a number in ones place (the 2), you get the same place value as the one being multiplied. That is, $2 \times .8$ will give tenths, $2 \times 5$ ones will give ones, and so on. When you multiply by a number in tens place (the 1), you get a place value one place higher. That is, $1 \times .8$ will give ones, $1 \times 5$ will give tens, and so on.

**1.** If you use what you know about place values in multiplication, what will be the decimal fraction in the product in this exercise? Why?

$$\begin{array}{r} 14.2 \\ \times\ 3 \\ \hline \end{array}$$

**2.** Will there be hundredths for the product in this exercise? Why?

$$\begin{array}{r} 126.24 \\ \times\ 32 \\ \hline \end{array}$$

**3.** Ones times tenths gives what place value?

**4.** Ones times hundredths gives what place value?

**5.** Tens times tenths gives what place value?

**6.** Tens times hundredths gives what place value?

## Multiplying a Decimal by a Whole Number

**1.** Multiply:

$$\begin{array}{r} 65.8 \\ \times\ 12 \\ \hline 131\,6 \\ 658 \\ \hline 789.6 \end{array}$$

$2 \times 8$ tenths gives 16 tenths. You now know that there should be tenths in the final product. (The decimal point does not need to be placed until the multiplications and additions are completed.) Explain the rest of the work.

When you multiply the 6 hundredths by the 4, what place value will you get for the product? Why does this give you the place of the decimal point in the final answer?

**2.** Multiply:

$$\begin{array}{r} 72.36 \\ \times\ 14 \\ \hline \end{array}$$

Multiplying 6 hundredths by the 1 in tens place gives what value? Finish the work.

**1.** Do you see that you do not have to keep the places of the multiplier in the same column as the number being multiplied? Tell why.

**a.** If you multiply tenths by a whole number, how many places should there be in the decimal-fraction part of the product? Why?

**b.** When you multiply a mixed decimal containing tenths by a whole number, how many decimal-fraction places will you have in the product? Why?

**c.** What value would you get if you were to multiply tenths by 10? hundredths by 10?

$$\begin{array}{r} 2.4 \\ \times\ 10 \\ \hline \end{array}$$

$$\begin{array}{r} 25.35 \\ \times\ 10 \\ \hline 253.50 \end{array}$$

Explain why 10 times any decimal-fraction place moves the decimal point in the product one place to the right of its original place in the number being multiplied. Do you have to read the place on the end of a decimal fraction if it is held by zero? Why?

**2.** Multiply:

| | | | | | |
|---|---|---|---|---|---|
| 42.3 | 2.34 | 14.6 | 32.65 | 64.05 | 132.25 |
| 14 | 16 | 10 | 100 | 12 | 64 |

## Multiplier Containing a Decimal Fraction

Suppose you want to multiply $0.8 \times 0.6$. Its meaning in common fractions is $\frac{8}{10} \times \frac{6}{10} = \frac{48}{100} = 0.48$. Why?

So, you would have hundredths when you multiply in decimal form.

| | |
|---|---|
| .6 | 4.6 |
| × .8 | × 3.2 |
| .48 | 92 |
| | 138 |
| | 14.72 |

What is the situation in the exercise $4.5 \times 5.27 = ?$ The smallest fractional part is seen to be $\frac{5}{10} \times \frac{27}{100}$, or $\frac{5}{10}$ of $\frac{7}{100}$. Why? This tells you what the name of the last place in the product should be. The product of $\frac{5}{10} \times \frac{7}{100} = \frac{35}{1000} =$ .035. Why?

These examples show you that tenths of tenths = hundredths and tenths of hundredths = thousandths. Does this help you understand the following general rule for multiplying decimals?

Multiplying hundredths by tenths gives thousandths. Why?

| | |
|---|---|
| 5.27 | 15.41 |
| 4.5 | 2.6 |
| 2 635 | 9 246 |
| 21 08 | 30 82 |
| 23.715 | 40.066 |

> When multiplying decimals, the number of places in the decimal-fraction part of the product is equal to the sum of the decimal-fraction places in the two numbers being multiplied.

**1.** How many decimal places in the product of each of these exercises?

| **a.** | **b.** | **c.** | **d.** | **e.** | **f.** |
|---|---|---|---|---|---|
| 46.3 | 68.42 | 831.65 | 974.63 | 83.01 | 68.55 |
| 27.1 | 23.6 | 73.8 | 421.5 | 97.6 | 42.7 |

**2.** Why can you drop the zero in the last place of a decimal fraction? (7.40 = 7.4, 2.450 = 2.45)

## Practice Problems and Exercises

**1.** The sixth-grade girls' club is buying a club pin for each girl. How much will 18 pins cost at $.31 each?

**2.** Jim's father said Jim could use .3 of the garden for his own use this spring. If the garden was 425 feet long, how much of this length was his share?

**3.** The contractor for the new school had 12 trucks, each holding 4.7 cubic yards of dirt. How many cubic yards could the trucks haul altogether if each truck hauled one load?

**4.** Clear Lake Camp said they would give some sixth-grade children room and board for a whole week at $7.85 a person. How much would it cost for 28 children?

**5.** If Ted batted .420 one year, how many hits did he get if he was at bat 200 times ($.420 \times 200$)?

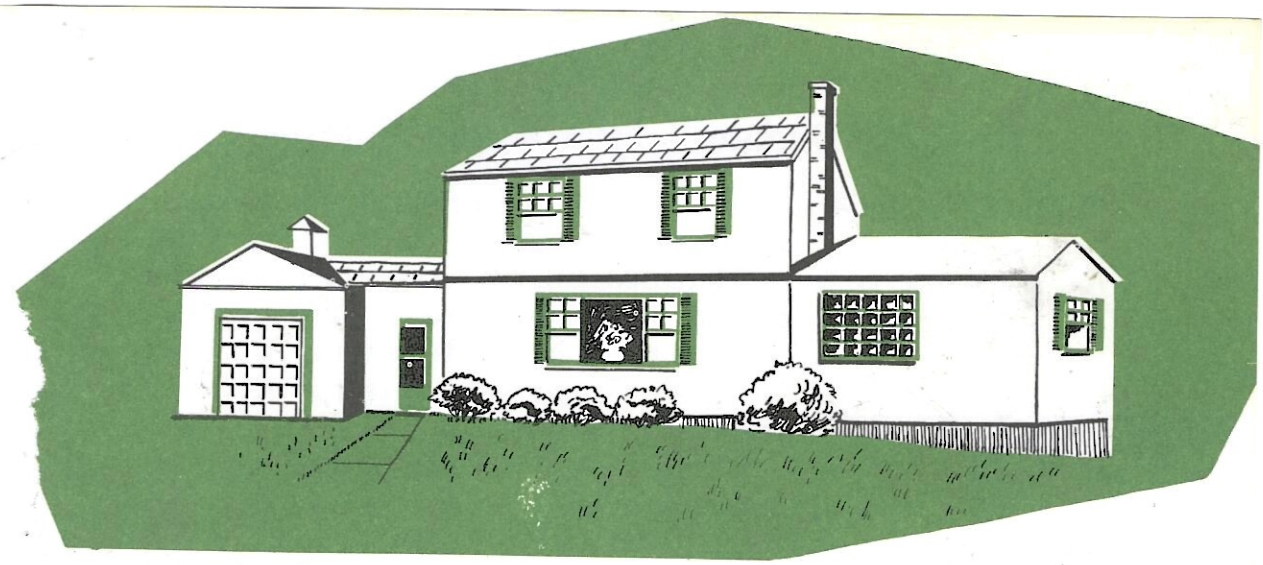
**6.** The art class wanted to buy a large quantity of colored paper to make posters for a local service organization. They were told that they could get it for $.006 per sheet, if they bought 1000 sheets. What would be the cost for this many sheets?

**7.** Scout uniforms cost $18.75 each. How much would 10 cost?

**8.** How much would Kathy spend on movies if she went to 100 movies a year and paid $.31 each time?

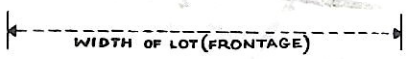
Multiply:

| | | | |
|---|---|---|---|
| **9.** 27.6 × 31 | **10.** 49.5 × 67 | **11.** 38.27 × 43 | **12.** 92.76 × 38 |
| **13.** 86.43 × 49 | **14.** 921.68 × 42 | **15.** 754.68 × 121 | **16.** 436.87 × 243 |
| **17.** 8.634 × 27 | **18.** 6.248 × 12 | **19.** 8.767 × 41 | **20.** 78.346 × 57 |
| **21.** 67.32 × 100 | **22.** 7.643 × 10 | **23.** 879.2 × 1000 | **24.** 3.465 × 100 |



## Harry's New Home




This picture of Harry's new home shows what he is thinking about most of the time. You see, he will have some work to do in the yard in the summer.

The contractor that developed the land had 274.4 ft. along the street to divide into 4 lots. How wide is the lot on which Harry's new home stands? $4\overline{)274.4}$

What are the names of the places in this number that is to be divided into 4 equal parts?

Will there be as much as 100 ft. along the street in each of the 4 lots? Why?

What do you do in dividing numbers when the values in the given places of the dividend are not large enough to be divided evenly by the divisors?

If hundreds are divided by ones, what is the name of the place in the quotient? Why?

What word could be used in place of the following question marks?

**a.** Tens ÷ ones gives a quotient figure in ? place.

**b.** Hundreds ÷ tens gives a quotient figure in ? place.

**c.** Tens ÷ tens gives a quotient figure in ? place.

## Dividing Decimals by a Whole Number

Divide 274.4 by 4 to find the width of Harry's lot on the preceding page:

$$\begin{array}{r} 68.6 \\ 4\overline{)274.4} \\ 24 \\ \hline 34 \\ 32 \\ \hline 24 \\ 24 \\ \hline \end{array}$$

2 hundreds are not enough to give a value in the hundreds place in the quotient. Change the 2 hundreds to tens; you have 27 tens. 27 tens ÷ 4 = 6 tens, with 3 tens remaining. Why?

34 ones ÷ 4 = 8 ones, with 2 ones remaining. Why?

You have now used all the whole numbers possible, so you must place a decimal point in your quotient to show that, from this point on, the values will be fractional.

You have 24 tenths to be divided. Why?

24 tenths ÷ 4 = 6 tenths with no remainder. Do these 6 tenths show that your decimal point was correctly placed? Harry's lot has 68.6 ft. frontage on the street.

Example: Divide 0.96 by 3:

$3\overline{).96}$

$$\begin{array}{r} .32 \\ 3\overline{).96} \\ 9 \\ \hline 6 \\ 6 \\ \hline \end{array}$$

There are no whole units to be divided, so the decimal point in the quotient can be placed above the point in the dividend. Why? Does 9 tenths ÷ 3 = 3 tenths? Does 6 hundredths ÷ 3 = 2 hundredths? Do you have to think of the place for the hundredths, or does this value always follow tenths?

**1.** If a contractor had a plot with a total frontage of 440.4 ft. and he wanted 6 equal lots, what would be the width of each lot?

**2.** What would be the width of each lot if there were to be 6 in a total frontage of 493.8 ft.?

Divide:

**3.** $4\overline{)14.4}$ $8\overline{)51.2}$ $24\overline{)57.6}$ $16\overline{)40.96}$ $12\overline{)1.44}$ $4\overline{)84}$

**4.** $5\overline{)76.15}$ $12\overline{)280.8}$ $23\overline{)1016.6}$ $31\overline{)1419.8}$ $6\overline{)1.26}$



## Taking a Trip by Car

**1.** The vacation record kept by Mr. Drew's family showed the following distances driven: 1st day—343.7 miles; 2nd day—352.4; 3rd and 4th days visited in Chicago; 5th day—346.9 miles; 6th day—197.3 miles; 7th day—493.6 miles; 8th, 9th, and 10th day visited in Denver; 11th day—573.1 miles; twelfth day—322.8 miles; 13th day—471.7 miles.

**a.** How many miles did the vacationers drive?

**b.** What was their average day's drive in miles for the whole trip (average = total distance divided by the number of days)?

**c.** What was their average for only those days that they were driving?

**2.** The following table gives the information about another family's vacation trip:

| Place | Mileage Reading | Gasoline (Gal.) Bought |
|---|---|---|
| Home | 3,462.4 | None |
| Toronto | 3,611.7 | 15.0 |
| Port Huron | 3,794.2 | 10.8 |
| Grand Rapids | 3,969.8 | 9.7 |
| Chicago | 4,144.2 | 11.0 |
| Davenport | 4,315.5 | 10.3 |
| Des Moines | 4,492.3 | 9.9 |
| Lincoln | 4,689.8 | 10.9 |
| Road Station | 4,928.5 | 12.8 |
| Denver | 5,184.3 | 13.6 |

How many miles did they average per gallon of gas during the trip to Denver?

## Jim Wants to Be Batting Champ

**1.** Jim is one of the leading hitters on his baseball team. In the last game he made 3 hits in his 5 trips to the plate. What was his batting average for this game?

To find a batting average, you divide the number of hits by the number of times at bat:

$5\overline{)3.0}$

$$\begin{array}{r} .6 \\ 5\overline{)3.0} \\ \underline{30} \end{array}$$

3 cannot be divided by 5 evenly. So 3 should be changed to 30 tenths in your thinking.

How do you get 3 ones = 30 tenths?

30 tenths ÷ 5 = 6 tenths. Why?

0.6 = 0.60 = 0.600. Why?

In baseball language, Jim would say he batted 600 for this game. It means that such an average is equal to 600 hits out of 1000 times at bat.



**2.** Jim made 15 hits out of 50 times at bat. He figured that he had a good chance of being batting champ. He found his average by dividing 15 by 50. 15 cannot be divided by 50 and give a whole number in the quotient. Why?

$50\overline{)15}$

$50\overline{)15.0}$

$$\begin{array}{r} .3 \\ 50\overline{)15.0} \\ \underline{150} \end{array}$$

You change the ones to tenths so they can be divided:

1 = 10 tenths,

15 = 150 tenths. Why?

150 tenths divided by 50 = 3 tenths.

How do you show that the 3 is tenths?

What is Jim's batting average (in baseball language) for these 15 hits out of 50 times at bat?

## The Size of Equal Parts

You know how to obtain the value, $\frac{5}{8}$ yd. of ribbon, by dividing 5 yd. into 8 equal parts. You can divide to change the $\frac{5}{8}$ into a decimal fraction.

In this problem the $\frac{5}{8}$ means the same as $5 \div 8$, or $8\overline{)5}$. Why?

$8\overline{)5 \text{ ones}}$

You know that it is impossible to divide 5 ones by 8 and get whole ones in the quotient. You must, therefore, change the 5 ones into smaller units. Why?

$8\overline{)50 \text{ tenths}}$

Try tenths: 5 ones = 50 tenths. $50 \div 8$ leaves a remainder. What is it?

$8\overline{)500 \text{ hundredths}}$

Try hundredths: 5 ones = 50 tenths = 500 hundredths. There will be a remainder here, too. Why?

$$8\overline{)5000}\quad 625$$

Try thousandths: 5 ones = 5000 thousandths. Why? Check this quotient.

You are dividing thousandths into 8 equal parts, so the quotient must be thousandths. $5 \div 8 = 625$ thousandths, or .625.

You should think this way:

$$\begin{array}{r} .625 \\ 8\overline{)5.000} \\ \underline{48} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \end{array}$$

5 = 5000 thousandths = 5.000 (read this as five thousand thousandths)

The zeros have changed the number 5 this way: One zero added changed the 5 to tenths, so 5 = 5.0 (read 5.0 as 50 tenths); two zeros added changed 5 to hundredths (read 5.00 as 500 hundredths); three zeros added changed 5 to thousandths (read 5.000 as 5000 thousandths)

Tell why the quotient is .625. Then 5 yd. $\div$ 8 = .625 yd.

Do you understand? When a number has zeros added on the right of the decimal point or on the right of a decimal fraction, the value remains unchanged. You have merely changed the units in the decimal value.

*Adding Zeros to a Decimal Value*

## What Can You Do with Zeros?

**1.** If 3 zeros are added to a number and the value remains the same, how are the units changed?

**2.** What happens to a number whose value must not be changed if you drop zeros on the right? For example, why is 4.50 = 4.5? 3.700 = 3.70? 3.70 = 3.7? \$3.00 = \$3?

**3.** Explain why 3.6 = 3.60 = 3.600.

**4.** Change $\frac{3}{8}$ to a decimal fraction by dividing the numerator by the denominator. (Find 3 decimal places.)

**5.** Are you using the idea of dropping the zero when you say that 350 ones equal 35 tens?

**6.** What happens to the units in a number when zeros are dropped from the end places?

**7.** Read these, using all the places:

**a.** 36.0 = 360 ? .
**b.** 2.50 = 250 ? .
**c.** 4.00 = 400 ? .
**d.** 5.270 = 5,270 ? .
**e.** 7.200 = 7,200 ? .
**f.** 6.000 = 6,000 ? .

**8.** Drop the end zero in each item in Exercise 7, and read the values.

**9.** Tell why these are equal:

**a.** 42.0 = 420 tenths = 4,200 hundredths.
**b.** 3.50 = 350 hundredths = 35 tenths.
**c.** 6.400 = 6,400 thousandths = 640 hundredths.
**d.** 3.140 = 3,140 thousandths = 314 hundredths.

**10.** Does dropping a zero on the end of a decimal fraction change its value? Does it change the units? Why?

**11.** Change to decimal fractions by dividing the numerator by the denominator:

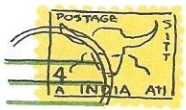
$\frac{3}{4}$ $\frac{1}{8}$ $\frac{1}{4}$ $\frac{7}{8}$ $\frac{3}{5}$ $\frac{8}{5}$ $\frac{9}{4}$ $\frac{5}{8}$ $\frac{11}{2}$ $\frac{9}{8}$

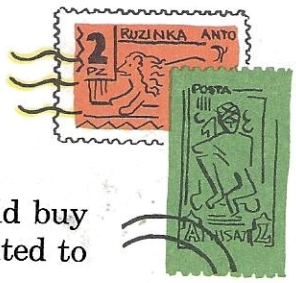
You will find it useful to memorize these equal values:

$\frac{1}{4} = 0.25$ $\frac{3}{4} = 0.75$ $\frac{2}{5} = 0.4$ $\frac{3}{5} = 0.6$ $\frac{4}{5} = 0.8$
$\frac{1}{8} = 0.125$ $\frac{3}{8} = 0.375$ $\frac{5}{8} = 0.625$ $\frac{7}{8} = 0.875$










# Dividing by 10 and 100

**1.** Frank found a stamp catalog that said he could buy a packet of 100 assorted stamps for $3.00. He wanted to know the average price per stamp. $100\overline{)\$3.00}$

$$\begin{array}{r} \$\ .03 \\ 100\overline{)\$3.00} \\ \underline{3.00} \end{array}$$

Why must you change the 3 to hundredths in order to divide?

Why does 300 hundredths ÷ 100 = 3 hundredths? Frank found that the stamps at the price of $3.00 per hundred would cost him 3¢ each. (Why does $.03 = 3¢?)

**2.** If a packet of 10 valuable stamps was $4.00, what would be the average cost of each stamp? $10\overline{)\$4.00}$

Were you keeping your eyes open as you examined these last 2 examples? Did you find a rule at work? Let's look again:

$10\overline{)4.0}$ or $4 \div 10 = 0.4$

$100\overline{)3}$ or $3 \div 100 = 0.03$

> If you divide a number by 10, the decimal point of the quotient is found to be one place to the left of its position in the number being divided.

How many places is it moved if you divide by 100?

Is this the opposite of where the decimal point is placed when you multiply numbers by 10 and 100? Explain.

**3.** Divide the following by 10:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| .37 | 2.96 | 42.25 | 45 | 6.90 | 167.50 | 450 | 600 |
| 4. | 40.04 | 34.97 | 5.741 | 965.3 | 1040 | 67.34 | 893.1 |

Divide the following by 100:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **5.** | 36.85 | 924.5 | 7.763 | 4700 | 943.01 | 6.935 | 48.67 |
| **6.** | 50.08 | 6.745 | 9.632 | 3261.5 | 8.764 | 9740 | 63.29 |





## Unit Test

**1.** The school Girls' Club charged \$.75 per month for dues. There were 23 members in the club. How much did they collect each month?

**2.** Freddy Hutchins allowed 82 runs one season in the 24 full games that he pitched. Find the earned-run average to the nearest hundredth.

**3.** Can you afford to buy the bike you want if it is advertised at \$63.00? The ad says it can be bought in 36 weekly payments with no down payment or carrying charges. If you have \$2.75 per week spending money, can you afford to buy this bike?

**4.** During the past 3 weeks, Mary earned the following amounts baby sitting: \$1.75, \$1.00, \$1.25, \$1.75, and \$1.50. She spent a total of \$.80 for carfare. What was her total profit?

**5.** Bill has put \$1.35 in a vacation savings club each week. If he continues to put the same amount in the club each week for a total of 50 weeks, how much money will he have to spend on his vacation?

**6.** Mary and her family took a trip. They drove 155.2 miles and used 8 gallons of gas. How many miles per gallon of gas did they average on this trip?

**7.** $25.6 \times 10$

**8.** $27.235 + 14.217$

**9.** $262.743 - 181.645$

**10.** $347.98 \times 76.4$

**11.** $37\overline{)2180.4}$

**12.** $100\overline{)2827.2}$

**13.** The Empire State Building is the tallest building in the world. It has 102 floors and is 1250 feet high. How high is each floor? (Give your answer as a mixed decimal with two decimal places.)

**14.** Write as decimals in hundredths:

$\frac{3}{2}$ $\frac{2}{5}$ $\frac{1}{4}$ $\frac{3}{8}$ $2\frac{1}{2}$ $\frac{7}{20}$ $\frac{4}{25}$

## Unit Review

**1.** If .7 equals 7 parts out of ten, what does .70 equal?

**2.** Johnny offered his brother Jimmy either $.1 or $.10. Which should he choose?

**3.** Which is larger, .5 or .51? How much larger?

**4.** Is .39 larger or smaller than .40? By how much?

**5.** Jan lived 2.7 mi. from school, while Sally lived 2.6 mi. away. Who had the longer distance to travel? How much longer?

**6.** In one store, a yard of ribbon cost $1.37, while in another store the same ribbon cost $1.40. From which store would you buy it? Why?

**7.** Does 0.5 of a basketball game take the same amount of time to play as 0.5 of a football game? Explain.

**8.** Which is larger, 2.33 or $2\frac{1}{3}$? How do you know?

**9.** Is there a difference between $2\frac{3}{8}$ and 2.3? What is it?

**10.** Name the place values:

42.5 78.3 861.2 927.58 .360 4.27 3.175

**11.** Add:

| | | | |
|---|---|---|---|
| 8.403 | 92.75 | 29.381 | 21.02 |
| 90.008 | 32.209 | 1.762 | 4.938 |

**12.** Subtract:

| | | | |
|---|---|---|---|
| 27.96 | 20.272 | 26.743 | 6.4 |
| 2.46 | 4.334 | 20.272 | 3.375 |

**13.** Write as decimals:

**a.** Twenty-eight and seventy-four hundredths.

**b.** Sixteen and nine hundredths.

**c.** Nineteen and two-hundred-thirty thousandths.

**d.** Four hundred and sixteen thousandths.

**14.** $46.2 \times 24$ $32.25 \times 2.5$

**15.** $25\overline{)160}$ $40\overline{)196}$ $2.8\overline{)23.52}$

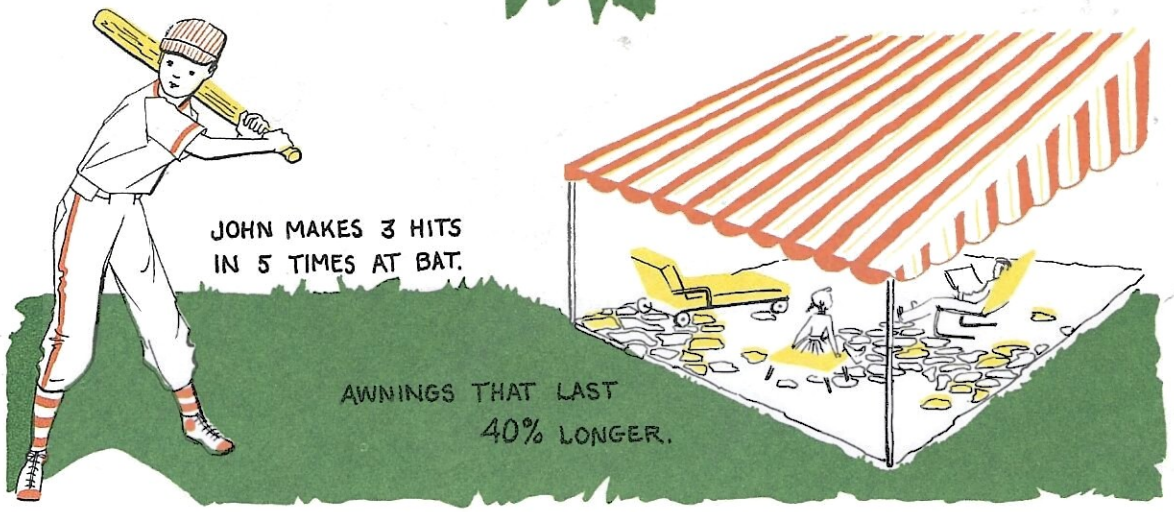
**16.** Change to decimals:

$\frac{7}{10}$ $\frac{3}{5}$ $\frac{1}{4}$ $\frac{7}{25}$ $\frac{27}{50}$ $\frac{15}{4}$ $\frac{23}{40}$

# UNIT 9

## Learning to Use Per Cent



You have learned how to express the relationship between two quantities by means of numbers. You first learned how to use whole numbers to measure these relationships. Then you learned how to make use of both common and decimal fractions when considering parts. Which of these ways are you using when you say:

**a.** One value is twice another?
**b.** One value is so much more than another?
**c.** You are one-half as old as someone else?
**d.** The rainfall this month was 0.4 of what it was last month?

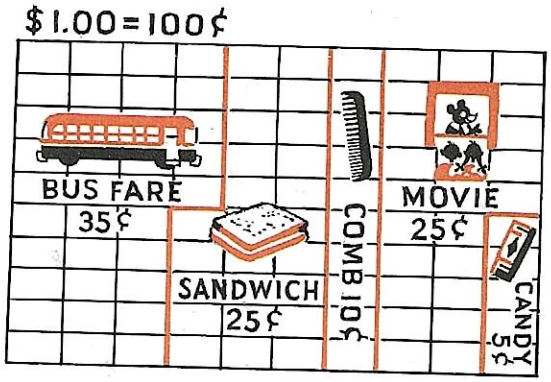
Many people, especially in the business world and in the scientific world, have another way to express the relations which they use in their work. They figure by hundreds, or on a hundredth basis. That is, they work in terms of *so many* for *every hundred*. This is called **percentage.**

Percentage is not a modern process at all. LaSalle, a Frenchman who had a Canadian settlement in the seventeenth century, ground grain for his subjects at his mill and took "6 measures out of every 100 measures" for his pay.

Do you think that the idea of so many out of a hundred seems to be a simple method of giving value to parts? You see examples of this in the picture on the opposite page.

**a.** A family can afford to pay $30.00 out of each $100 of income for rent.
**b.** Some cows produce 10 lb. of cream in every 100 lb. of raw milk.

**1.** John makes 3 hits in 5 times at bat. At this rate, how many hits would he make in 100 times at bat?
**2.** 5 things out of 20 is the same as ? out of 100.
**3.** A tenant paid the rent on a field of wheat by giving the owner 2 out of every 5 bu. that he raised. How many out of every 100 bu. would be the amount of rent?




## Sally Spends One Dollar

Sally needed to go shopping one Saturday. She decided that she would take the bus, buy a sandwich when she arrived in town, do some window shopping, buy the things she needed, and go to a movie in the afternoon.

Sally had only a dollar bill. She had to figure how she was going to spend it. The chart shows her budgeting. If the total square stands for the dollar or 100¢, for what value will a small square stand?

Do you see that she spent 35¢ out of 100¢ for busfare, or $\frac{35}{100}$ of what she had to spend?

**1.** How many cents out of the 100¢ did she spend on each item?

**2.** Could you say that she spent $\frac{1}{4}$ of a dollar for her sandwich? Why? Does $\frac{1}{4} = \frac{25}{100}$?

**3.** Sally's busfare, sandwich, comb, and candy bar would cost her 75¢. Why is 75¢ the same as $\frac{3}{4}$ of a dollar? Does $\frac{3}{4} = \frac{75}{100}$? Why?

**4.** Explain the meaning of the figures:

$$\$\tfrac{3}{4} = 4\overline{)3.00}^{.75} = \tfrac{75}{100} = 75¢$$

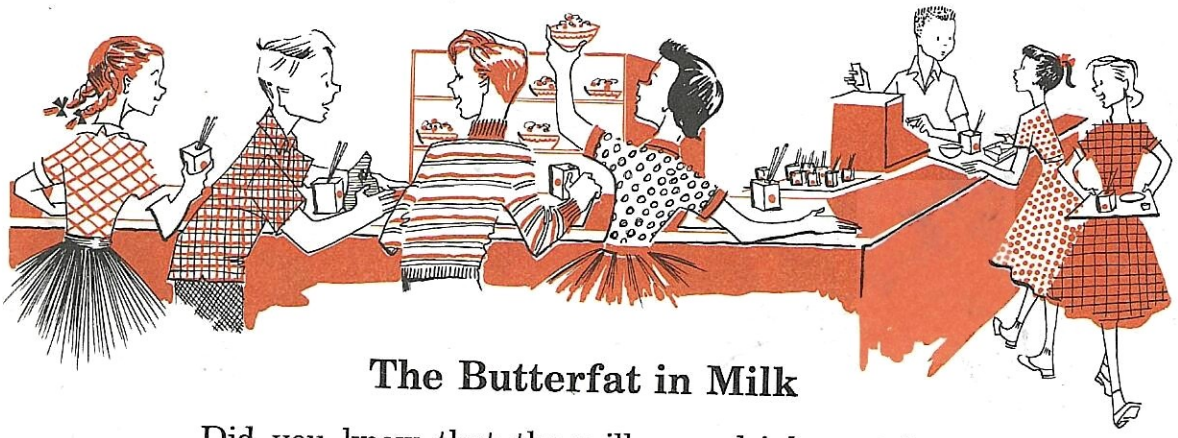
**5.** Use division to show that $\frac{3}{5}$ is the same value as 60 parts out of 100 parts, or the ratio 60 to 100.

**6.** To compare two fractional values, your old method was to use a common denominator. Look at these values to see how figuring by the hundred is an easy way to compare $\frac{3}{4}$ and $\frac{4}{5}$:

$$\tfrac{3}{4} = \tfrac{15}{20} \qquad \tfrac{3}{4} = \tfrac{75}{100}$$
$$\tfrac{4}{5} = \tfrac{16}{20} \qquad \tfrac{4}{5} = \tfrac{80}{100}$$

How could you change $\frac{15}{20}$ to $\frac{75}{100}$ or $\frac{16}{20}$ to $\frac{80}{100}$?

$\frac{3}{4}$ compares with $\frac{4}{5}$ in the same way that 75 compares with 80 (on the hundredth basis).



## The Butterfat in Milk

Did you know that the milk you drink contains approximately 4 parts butterfat in 100 parts of milk? But you don't see these figures on the caps of the bottles of milk which you buy.

The value which tells the specified number of parts in every hundred is called the *per cent.* It comes from the Latin *per*, meaning by, and *centum*, meaning hundred. The symbol for per cent is %. 4 out of 100 parts would be 4 per cent, or 4%.

If the milk contained 5 parts butterfat in 100 parts of milk, the per cent of butterfat would be ?.

**1.** A reindeer may produce milk that is 17% butterfat. What does this mean when measuring by the hundred?

**2.** A herd of goats might produce milk that is 8% butterfat. How many pounds of butterfat are there in 100 pounds of the milk produced by the herd?

**3.** Explain these figures:

**a.** $\frac{2}{5} = \frac{4}{10} = \frac{40}{100} = 40\%$

**b.** $\frac{2}{5} = 5\overline{)2.0} = .40 = 40\%$

Both equal a ratio of 2 to 5. Why?

Fill the blanks:

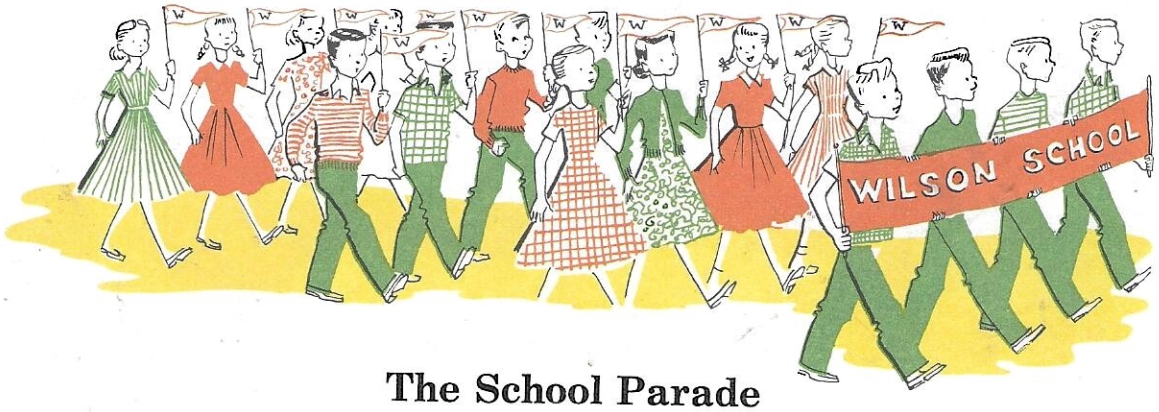
**4.** 15 out of 100 parts = $\frac{15}{100}$ = ?%, or the ratio $\frac{3}{20}$. Why?

**5.** Why is a percent a way of writing a ratio?

**6.** What per cent of a dollar is 5¢? $.01?

**7.** 25% = ? out of ? , or $\frac{?}{100}$.

**8.** .25 = ?%; .8 = $\frac{?}{100}$ = ?%; .65 = ?%; 6% = $\frac{?}{100}$.




## The School Parade

The children in the Wilson School were to march in a parade on Saturday. 8 out of 32 pupils in Miss Johnson's class could not attend. What fractional part of the class could not attend? What per cent could attend?

$$\begin{array}{r} 32 \\ -\ 8 \\ \hline 24 \end{array}$$

24 children could attend.
This is $\frac{24}{32} = \frac{3}{4}$ of the class. Why?

The $\frac{3}{4}$ means 3 parts out of 4 parts. How many out of 100 would be the same relationship as 3 parts out of 4 parts? $\frac{3}{4} = \frac{?}{100}$.

$\frac{3}{4} = \frac{75}{100} = 75\%$ Why? 100 | $\frac{1}{4}$ PART | $\frac{1}{4}$ PART | $\frac{1}{4}$ PART | $\frac{1}{4}$ PART

1. If 8 pupils out of 40 were absent, what % attended?
2. If 12 pupils out of 30 were absent, what % attended?
3. If 12 pupils out of 36 were absent, what % attended?

Use the fraction form to find these per cents:

4. $\frac{3}{5} = \frac{?}{100} =$ ? % $\quad \frac{2}{3} = \frac{?}{100} =$ ? % $\quad \frac{3}{10} = \frac{?}{100} =$ ? %
5. $\frac{3}{8} = \frac{?}{100} =$ ? % $\quad \frac{8}{10} = \frac{?}{100} =$ ? % $\quad \frac{5}{6} = \frac{?}{100} =$ ? %
6. Change to per cents:

$\frac{9}{10}$ $\quad \frac{3}{5}$ $\quad \frac{2}{8}$ $\quad \frac{4}{10}$ $\quad \frac{6}{8}$ $\quad \frac{4}{5}$ $\quad \frac{8}{10}$ $\quad \frac{4}{8}$

7. On the last day before a vacation, 32 out of a class of 40 were present. What % were present? What ratio?

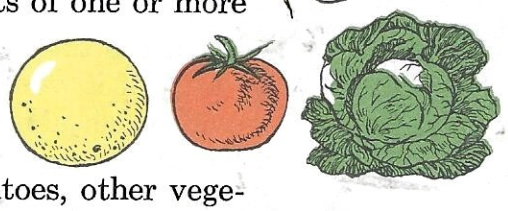
## The Seven Basic Foods

In order for you to be as healthy as possible, those who prepare your food have to plan very carefully. One of the important rules a mother follows, as far as possible, is to serve the proper amounts of the 7 basic foods.

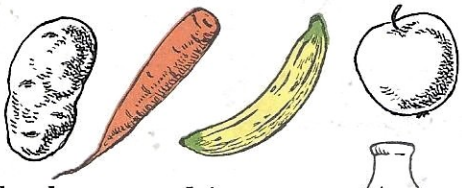
**1.** Everyone should have one or more servings daily of the first basic food group, which is made up of leafy green and yellow vegetables. If you were to count all the food you eat in a day as 100 parts, this group should make up about 14 parts or $\frac{14}{100}$ or .14 or 14%.



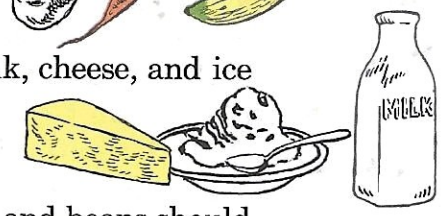
**2.** The second basic food group consists of one or more helpings of citrus fruits, tomatoes, or raw cabbage. This group should make up about 14 parts out of 100 of your daily diet. What per cent is this?



**3.** The third group is made up of potatoes, other vegetables, and fruit. You should probably get 2 or more servings from this group for your best diet. You should have about 22 out of 100 parts of what you eat daily. What per cent is this?



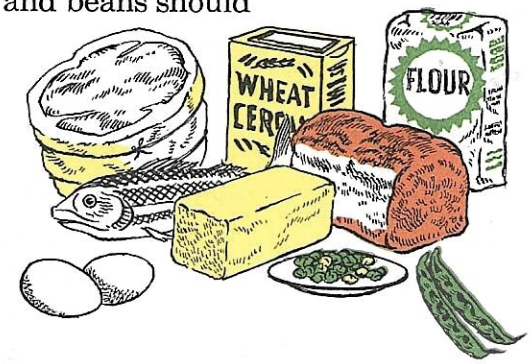
**4.** The fourth group is made up of milk, cheese, and ice cream. It makes up about 28 parts out of 100 parts of a well balanced diet. Give the per cent for this group.

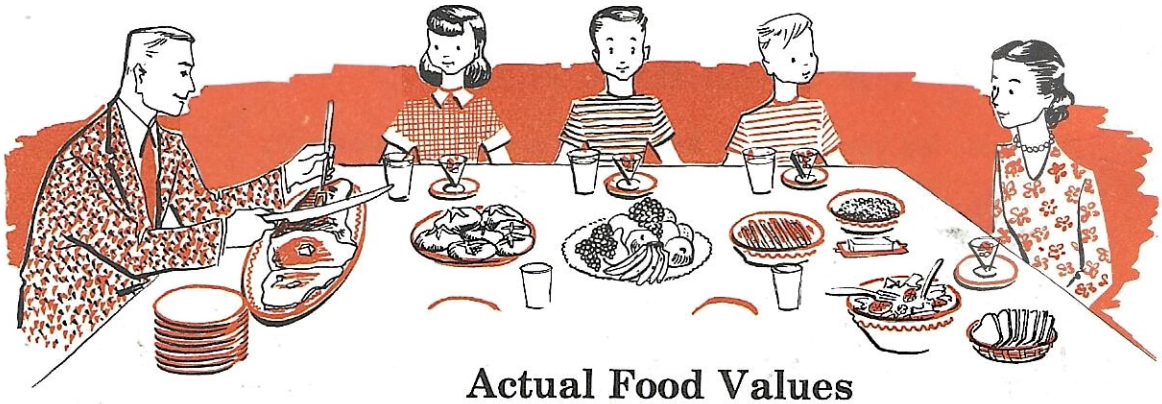



**5.** Meat, poultry, fish, eggs, dried peas and beans should give you about 12 out of 100 parts of what you eat each day. $\frac{12}{100} = ?\ \%$.

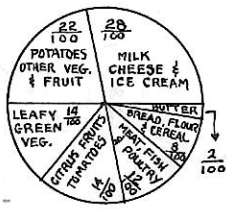
**6.** Bread, flour, and cereal should be about 8 parts out of your total of 100 parts. $\frac{8}{100} = ?\ \%$.

**7.** You should eat about 2 parts of your total from butter or fortified margarine. $\frac{2}{100} = ?\ \%$.





## Actual Food Values




When your mother is planning a balanced diet for you, she has to know the food values of the various things in each of the groups on the last page. See the graph here.

**1.** In the total amount of food you eat each week, $\frac{14}{100}$ or 14% of it would weigh about 3 lb. Then, $\frac{21}{100}$ or 21% would weigh about $1\frac{1}{2}$ times as much, or $4\frac{1}{2}$ lb. At this rate, how many pounds of milk, cheese, and ice cream should you have each week?

**2.** About how much leafy green vegetables should you expect to eat in a week (total amount, about 21 lb.)?

**3.** Change the fractions in the graph to per cents. Add them to see if they total 100%. Why does 100% mean $\frac{100}{100}$ or all of the parts of any value?

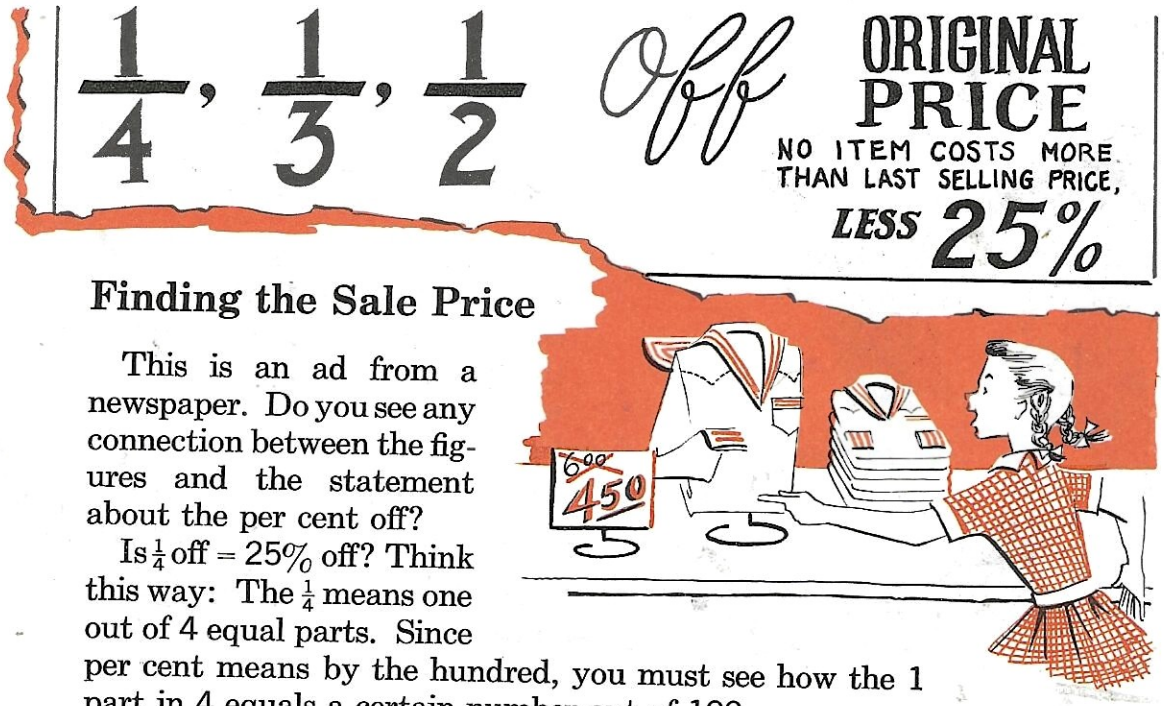
**4.** The total of any one kind of basic foods you consume in 1 day can be 100%. Why?

**5.** If 1 cup of milk will supply 30% of your daily calcium needs, how much is needed to supply 100% of them?

**6.** One-half cup of leafy green and yellow vegetables will supply about 25% of your daily vitamin needs. How much is needed to supply all of them?

**7.** One egg will supply 10% of your daily iron needs and 1 slice of bread will supply 5%. Tell different ways in which bread combined with eggs will supply 100%.

**8.** Four oz. of meat, poultry, or fish will supply 20% of your daily protein needs. How many ounces of this group of foods should you eat each day?




## Finding the Sale Price

This is an ad from a newspaper. Do you see any connection between the figures and the statement about the per cent off?

Is $\frac{1}{4}$ off = 25% off? Think this way: The $\frac{1}{4}$ means one out of 4 equal parts. Since per cent means by the hundred, you must see how the 1 part in 4 equals a certain number out of 100.

The whole must be divided into 100 equal parts:

| 100 PARTS | | | |
|---|---|---|---|
| ONE PART | ONE PART | ONE PART | ONE PART |
| 25 | 25 | 25 | 25 |

Then, the one out of four would be 25 out of one hundred. Thus, $\frac{1}{4} = \frac{25}{100} = 25\%$.

**1.** Does $\frac{1}{4}$ off \$6.00 = \$4.50? ($\frac{1}{4}$ of \$6.00 = \$1.50 Why?)

**2.** $\frac{1}{2}$ = what per cent?

**3.** Figure the sale price of the following, assuming that they were advertised at $\frac{1}{5}$ off:

Raincoats \$20.00; Dresses \$10.00; Rifle \$30.00.

What per cent reduction is equal to $\frac{1}{5}$ off?

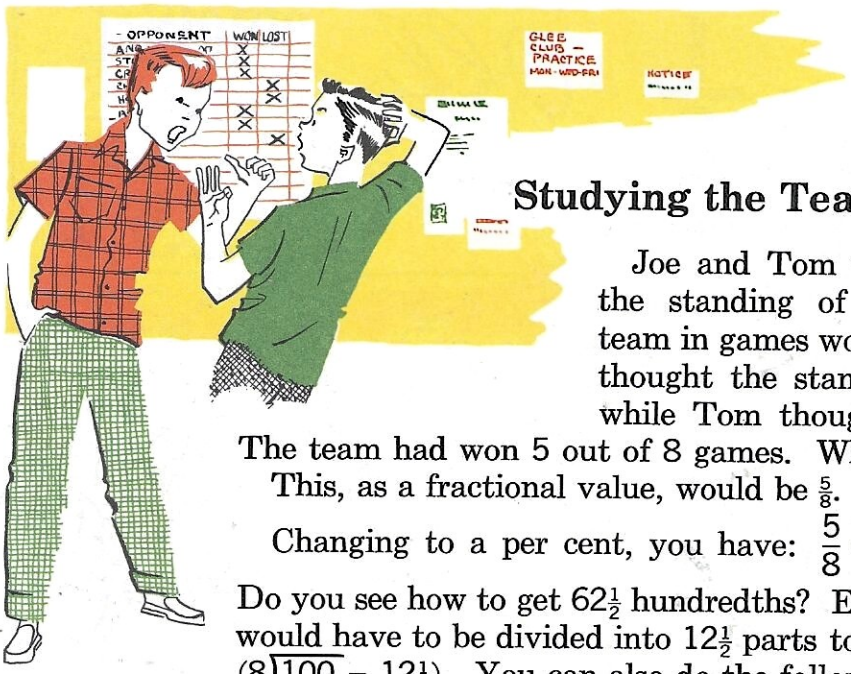
**4.** You bought an article at $\frac{1}{3}$ off. What was the per cent of reduction?

$\frac{1}{3} = 3\overline{)1.00}^{.33\frac{1}{3}} = ?\%$

**5.** An article sells for $\frac{9}{10}$ of its original price. What per cent tells the amount of reduction?

$\frac{10}{10} - \frac{9}{10} = \frac{1}{10}$ Why?

$10\overline{)1.00}^{.10} = ?\%$




## Studying the Team Standing

Joe and Tom were discussing the standing of their baseball team in games won and lost. Joe thought the standing was poor, while Tom thought it was fair. The team had won 5 out of 8 games. Who was right?

This, as a fractional value, would be $\frac{5}{8}$. Why?

Changing to a per cent, you have: $\frac{5}{8} = \frac{62\frac{1}{2}}{100} = 62\frac{1}{2}\%$.

Do you see how to get $62\frac{1}{2}$ hundredths? Each eighth part would have to be divided into $12\frac{1}{2}$ parts to give 100 parts ($8\overline{)100} = 12\frac{1}{2}$). You can also do the following:

$\frac{5}{8} = 5 \div 8$

$$8\overline{)5.00} = .62\tfrac{1}{2}$$

48

20

16

4 R

$4 \div 8 = \frac{1}{2}$

Divide the numerator by the denominator. Change the 5 ones to 500 hundredths. You want hundredths because they give you the per cent value. The team's wins were $62\frac{1}{2}\%$ of the games played.

This is how to change a fraction like $\frac{3}{8}$ to a per cent:

$$\tfrac{3}{8} = 8\overline{)3.000} = .375 = 37.5\%$$

Per cents are the equivalents of hundredths. So, reading the places in .375, you see that you have 3 tenths, 7 hundredths, and 5 thousandths. You can also read it as 37 hundredths and 5 tenths of a hundredth. Why?

Thus, .375 = 37.5 hundredths, or 37.5%.

Do you understand why moving the point 2 places to the right changes a decimal to a per cent?

## Practice Problems and Exercises

**1.** Your team lost 3 games out of 15. What % did you win? The ratio of wins to total games is ?.

**2.** If there are 4 ounces of pigment to 10 ounces of paint, what % is the pigment?

**3.** In a sale you are given a discount of 15¢ on a 50¢ item. What % do you save?

**4.** There are 40 children in Miss Johnson's class, but 8 are absent. What per cent is present?

**5.** The Red Cedar School was having a contest to see which room could have the most children selling 4 tickets each for the Spring Carnival. In the 6th grade, 23 out of 25 sold their 4 tickets. In the 7th grade, 36 out of 40 sold their 4 tickets. Which room had the better record? How much better in per cent?

**6.** A certain blanket contained 27 oz. of wool, 45 oz. of cotton, and 18 oz. of nylon. What per cent of each does it contain?

**7.** A kilometer is approximately $\frac{5}{8}$ of a mile. What per cent of a mile is it?

**8.** If you saved $1.25 on a $3.75 book, what per cent did you save?

**9.** Write as common fractions by the hundred, then write the per cents:

| | | | | |
|---|---|---|---|---|
| $\frac{1}{2} = ?\%$ | $\frac{1}{4} = ?\%$ | $\frac{1}{5} = ?\%$ | $\frac{3}{4} = ?\%$ | $\frac{1}{8} = ?\%$ |
| $\frac{1}{20} = ?\%$ | $\frac{1}{3} = ?\%$ | $\frac{1}{12} = ?\%$ | $\frac{1}{15} = ?\%$ | $\frac{9}{20} = ?\%$ |
| $\frac{3}{5} = ?\%$ | $\frac{1}{10} = ?\%$ | $\frac{7}{10} = ?\%$ | $\frac{2}{3} = ?\%$ | $\frac{3}{16} = ?\%$ |

**10.** Use division to find the decimal values in hundredths, then write as per cents:

| | | | | |
|---|---|---|---|---|
| $\frac{3}{20} = ?\%$ | $\frac{7}{12} = ?\%$ | $\frac{7}{20} = ?\%$ | $\frac{3}{4} = ?\%$ | $\frac{4}{5} = ?\%$ |
| $\frac{7}{8} = ?\%$ | $\frac{2}{5} = ?\%$ | $\frac{3}{8} = ?\%$ | $\frac{3}{10} = ?\%$ | $\frac{11}{16} = ?\%$ |

**11.** When you take $\frac{1}{2}$ of something, you are taking 50% of it. Why?

**12.** What per cent of a whole is the same as $\frac{2}{5}$ of it?

## Using Decimal Values for Per Cents

If you want to find 36% of 600 by using a fraction, this is the method:

$$\frac{36}{\cancel{100}} \times \overset{6}{\cancel{600}} = 216$$

You divide by 100. Then you multiply the quotient by the number in the per cent value. Why?

But you also know that $\frac{36}{100} = .36$.

Can you multiply 600 by .36? Let's see:

$$\begin{array}{r} 600 \\ .36 \\ \hline 36\,00 \\ 180\,0 \\ \hline 216.00 \end{array}$$

Can you always change a per cent value to a decimal?

Why do you point off 2 decimal places in the product? Can you read the product as 216? Why? Does the same method work if you want to find $\frac{3}{4}$ of some value?

$$\frac{3}{4} \times 800 = \frac{75}{100} \times 800 = .75 \times 800 = ?$$

**1.** Does per cent of mean hundredths times, in finding the size of a part?

**2.** Find 18% of 256:

$$\frac{18}{\cancel{100}} \times \overset{2.56}{\cancel{256}} = ? \quad \text{or} \quad .18 \times 256 = ?$$

Which method do you prefer?

**3.** Find 5% of \$1,275:

$$\begin{array}{r} \$1275 \\ \times\ .05 \\ \hline \end{array}$$

To find 5% of \$1275 you multiply 1275 by .05. Why? There are two decimal places in the numbers being multiplied. There must be 2 places in the product. Why?

**4.** Exercises in finding the per cent of a number:

50% of 168 = ? 25% of 160 = ? 30% of 120 = ?
45% of 260 = ? 15% of 230 = ? 35% of 175 = ?

**5.** Can you state a rule for finding the per cent of any number? Tell why the work given here is correct for 6% of \$102.40:

$$\frac{6}{\cancel{100}} \times \overset{1.024}{\cancel{102.40}} = \$6.144$$

$$\begin{array}{r} \$102.40 \\ .06 \\ \hline \$6.1440 \end{array}$$

## Finding the Per Cent of a Number

**1.** John's mother gets \$50.00 a week to cover the expenses of the house. She spends 40% of this weekly budget for food. How much money does she spend for food?

**2.** Alice's mother tries to save 10% of her money each week. How much would she save in a week if she received \$72.50?

Find out how much Alice's mother could save in a year at that rate.

**3.** Jack gets an allowance of \$2.20 a week. He saves 10% of this and buys saving stamps with the money. How much does he save? Find how many 10¢ stamps he can buy each week.

If he finds that he spends 20% of his allowance on candy and ice cream, how much of his money is spent for these items? Is this too much for candy?

In checking, Jack found that he spends 15% of his money for movies, if he attends only once a week. Find how much he spends for 1 movie.

**4.** Mary has been saving part of her allowance to buy her mother a pocketbook. She can save 25% on the cost by buying it now. If the former price was \$5.95, find how much the pocketbook will cost her (without tax). How much is the saving?

**5.** If Mary saves 35% of her \$2.00 a week allowance for the next 5 weeks, will she have enough money to buy the pocketbook in Problem 4?

**6.** Andy's father found he spent 29% of his last month's salary for clothing for the family. If his salary is \$320, find out how much of it went for clothing.

**7.** A farmer brought 755 pounds of milk to the dairy. The milk contained $3\frac{1}{2}$% butterfat. How many pounds of butterfat were in the milk that he delivered?

## Unit Review

Change the following fractions to per cents:

1. **a.** $\frac{2}{5} = ?\%$ **b.** $\frac{3}{8} = ?\%$ **c.** $\frac{5}{6} = ?\%$ **d.** $\frac{7}{16} = ?\%$
2. **a.** $\frac{7}{10} = ?\%$ **b.** $\frac{3}{20} = ?\%$ **c.** $\frac{7}{12} = ?\%$ **d.** $\frac{5}{12} = ?\%$

Change the following per cents to common fractions:

3. **a.** $60\% = ?$ **b.** $55\% = ?$ **c.** $37\frac{1}{2}\% = ?$ **d.** $42\% = ?$
4. **a.** $12\% = ?$ **b.** $96\% = ?$ **c.** $25\% = ?$ **d.** $40\% = ?$

5. Express each of the following per cents as decimals:

$40\% = ?$ $65\% = ?$ $50\% = ?$ $83\% = ?$ $31\% = ?$
$66\frac{2}{3}\% = ?$ $87\frac{1}{2}\% = ?$ $62\frac{1}{2}\% = ?$ $8\% = ?$ $2\% = ?$

6. Find:

$\frac{36}{100}$ of 4000 32% of 480 6% of 379 $12\frac{1}{2}\%$ of 800

7. One gasoline manufacturer guarantees 12% more mileage to the user of his fuel, or your money back. If you had been averaging 20 mi. per gallon, how many additional miles should you expect when using this manufacturer's product?

8. What fractional part of the former price do you pay for something that is selling now at 20% off?

9. How much do you save on the cost of an article that sold formerly for $8.00 and is now selling at 25% off?

10. If 75% of your class bought Christmas seals this past year, how would you write this fact as a decimal?

11. How would you change 75% to a fraction?

12. Our attendance record for last month read $98\frac{1}{2}\%$. Write this in terms of a decimal.

13. 60% of our class are girls. One-tenth of the girls have long hair. Find the per cent of girls in the class who do not have long hair.

14. The boys' softball team has won 3 out of 4 of the games that were played. What is the per cent of wins?

15. The highest batting average of any one of our boys is 300. How would you give this value as a per cent?

## Unit Test

**1.** Find the per cent that is equal to these values:

$\frac{1}{4} = ?\%$ $\frac{1}{10} = ?\%$ $\frac{1}{5} = ?\%$ $\frac{1}{3} = ?\%$ $\frac{1}{8} = ?\%$ $\frac{1}{6} = ?\%$

$\frac{1}{12} = ?\%$ $\frac{9}{15} = ?\%$ $\frac{5}{8} = ?\%$ $\frac{11}{20} = ?\%$ $1\frac{1}{2} = ?\%$ $\frac{3}{10} = ?\%$

**2.** Write these decimal values as per cents:

.27 .46 .32 .97 .67 .57 .1 .7 .05 .2 .08 .9

**3.** If a bowling team won 4 out of 6 games, what per cent did it lose?

**4.** Mrs. Jones operates her household on a budget which allows 35% for food. If her total budget is $280 for the month, how much may she spend for food?

**5.** The morning newspaper hired Phil as a delivery boy. He missed delivering the paper only 11 out of 200 days. His sister Edith delivered the papers for him on the days he missed. What per cent of the time did Edith deliver Phil's papers?

**6.** A business enterprise failed and paid its creditors $.15 on the dollar. What per cent did it pay?

**7.** Johnny is given a 5¢ discount on the price of a screwdriver. If the price of the screwdriver is $1.00, what is his per cent of discount?

**8.** What per cent of interest would you be paying if you had to pay $5.00 on $100 that you borrowed for one year?

**9.** Our mayor received .57 of the votes cast in the last election. What per cent of the votes did he receive?

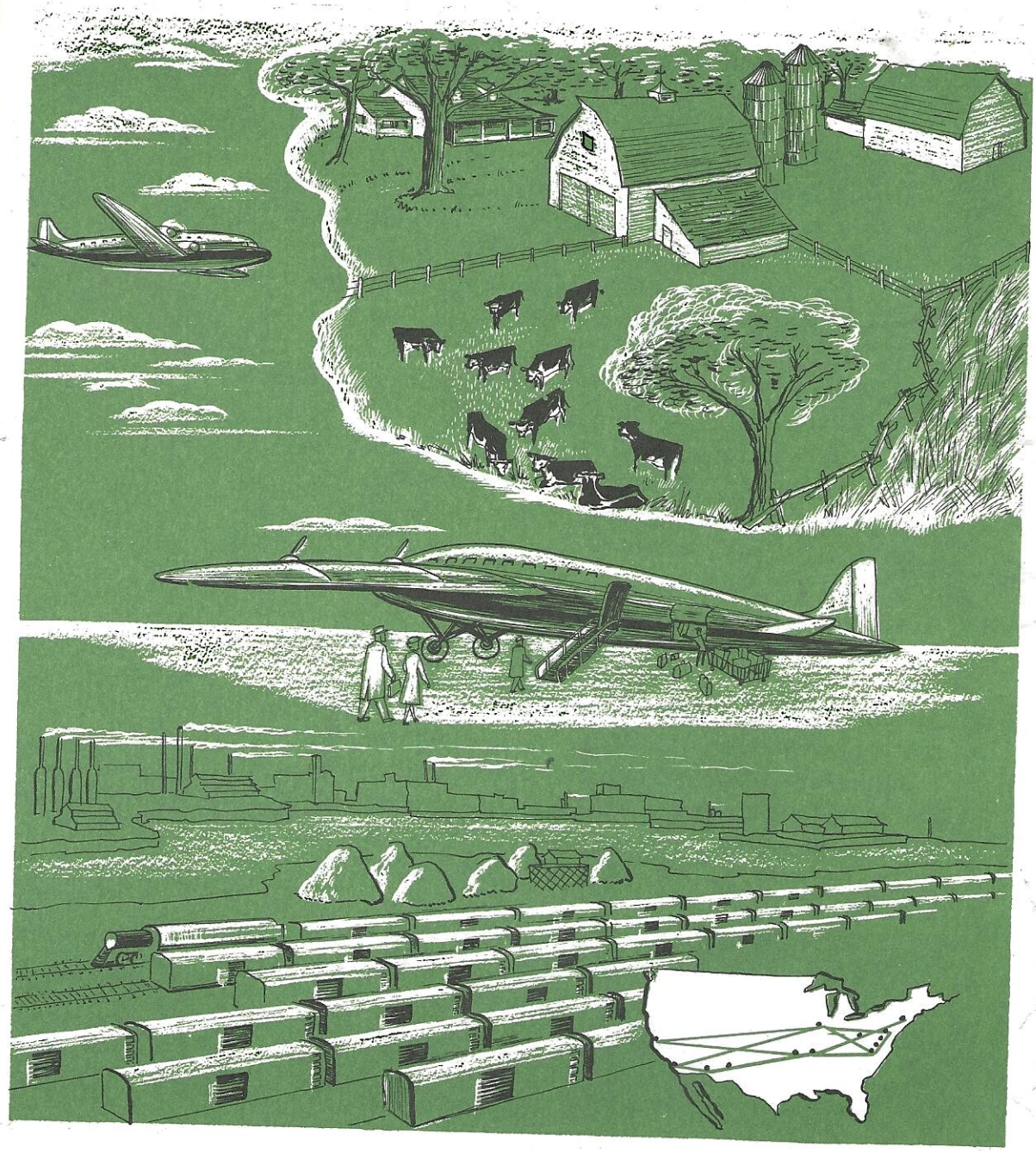
**10.** Jane had been planning to buy a new coat next fall. The local clothing store was having a special sale of fall and winter coats at 50% off. How much would Jane save by buying her coat at the sale if the original price was $27.50?

**11.** Verne wanted some high-top boots for next winter. The boots sold regularly for $15.50. How much would he save if they were on sale at 30% off?

**12.** Find 26% of 265.25.

# UNIT 10

# Seeing Quantity All Around You



Any one of the items in the picture on the opposite page shows examples of quantity. Take the airplane, for example. It has a definite number of engines. Its take-off speed is represented by quantity; also its landing speed, top speed, and cruising speed. It carries a definite quantity of gas on each trip. That amount has weight. The weight of the gas must be taken into consideration when deciding how many passengers and how much freight can be carried. The amount of gas that is used depends on the speed. There is a relationship between the amount used and the time the plane is in the air. A certain amount extra must be allowed for safety. The exact amount depends upon another quantity—the distance to a safe landing field. Do you know how many pounds of baggage you are permitted to take on an airplane flight?

The farm scene shows a herd of cattle. Each cow requires a certain amount of feed each day. A farmer needs to be able to answer questions like the following:

**a.** How many barns are needed to provide shelter for his herd?

**b.** How much seed is needed to sow an acre of land? 20 acres of land?

**c.** How much will the seed cost?

**d.** How much grain can he expect a certain amount of seed to yield?

**e.** How many hours will be needed to care for a crop?

**f.** What profit can he expect?

Trains are an important method of transporting both people and goods. The freight cars shown in the picture may travel from California to Maine and back again many times. A carload of each kind of goods weighs a certain number of tons. What is a ton? How far is it across the United States? If it costs $4.50 a ton in carload lots to ship coal a certain distance, what is the freight cost on a 40-ton load?

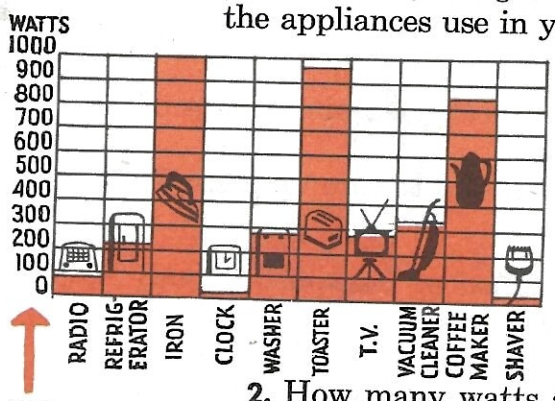
# Electricity in the Home

Modern inventions make daily living easier by helping to save time and energy. Among the industries that contribute toward making life more pleasant is the electrical industry. Here are some facts about the ten most popular electrical appliances:

A Recent Year's Estimates of the Kilowatt Hours of Electricity Used Yearly by Ten Popular Appliances

| Appliance | Yearly estimate | % homes wired | No. in use |
|---|---|---|---|
| Radio | 100 | 98.4 | 45,775,000 |
| Refrigerator | 350 | 90.4 | 39,397,000 |
| Iron | 115 | 90.1 | 39,272,500 |
| Clock | 17 | 83.7 | 36,470,000 |
| Washer | 45 | 78.5 | 34,194,000 |
| Toaster | 35 | 71.9 | 31,350,000 |
| TV | 280 | 63.1 | 27,500,000 |
| Vac. Cleaner | 25 | 60.5 | 26,370,000 |
| Coffee Makers | 80 | 54.3 | 23,677,800 |
| Shaver | 1 | 34.1 | 14,873,500 |

How many of these appliances do you have in your home? Can you figure from the chart how much current the appliances use in your home per year?




AVERAGE HOURLY WATTAGE USED BY 10 ELECTRICAL APPLIANCES

Can you read the bar graph on the left? It shows how many watts (a watt is a measure of the quantity of electricity) are used per hour by each appliance. A radio, for example, used approximately 90 watts per hour.

1. How many watts does a washer use per hour? a toaster?

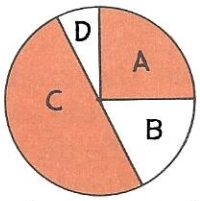
2. How many watts an hour does a TV set use? How many kilowatts a year does it use?

3. How many people in 100 have radios in their homes? How many have electric shavers? What approximately is the ratio of radios to shavers?

In addition to the appliances listed, there are many more. Electricity is used in many other ways. How many can you list now? Do you have any idea how much electricity is used in a year in the United States? There were 342,524,000,000 kilowatt hours sold in a recent year. Can you read this number?

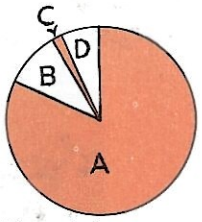
Here are the groups that bought electricity and also the per cent of buyers in each group. There was a total of 48,459,371 customers.

A. Residential—25%
B. Commercial—18%
C. Industrial—49%
D. Others—8%




Amount of Electricity Used

A. Residential—84%
B. Commercial—11½%
C. Industrial—½%
D. Others—4%




Number of Customers Using Electricity

**1.** Which group used the most electricity?

**2.** Which group had the most customers?

**3.** Why didn't the group that had the most customers use the most electricity?

**4.** Can you tell from the above figures how many residential customers there were in the year that is reported? Can you figure out how many kilowatt hours of electricity they used?

**5.** Can you show that you know the meaning of 25% by expressing it in another way? Can you think of still another?

**6.** Round off the number of refrigerators in use to the nearest 10 million. How many are in use?

**7.** How much electricity would all these refrigerators use in a year? What per cent of the total electricity used by residential customers would this be?

**8.** Would you know how to go about figuring out how much electricity a year was used by residential customers for all electrical appliances other than the top ten that are reported here?

## What Do the Numbers Mean?

The table on Page 296 contains values that are so great it is difficult to understand their real significance. Comparisons with other numbers may make their meaning clearer. For example, the estimated population in the U.S.A. in 1954 was 166,000,000. This is read as one hundred sixty-six million. If there were 46,000,000 radios (approximately), how many persons were there per radio?

$$
\begin{array}{r}
3 \\
46\overline{)166} \\
\underline{138} \\
28 \text{ R}
\end{array}
$$

Quotient: $3\frac{14}{23}$

Since the units are alike (millions), you can divide as shown.

Check the division.

Is the figure nearer to 4 persons per radio than 3?

**1.** There is about 1 electric shaver to every 11 persons. Is this estimate accurate?

**2.** What is the difference between the number of electric washers and electric clocks in use?

**3.** A kilowatt is the measure of the amount of electric current used. Which appliance uses the most per year?

**4.** Do you know why the electric iron uses so much current each hour while in use (see the graph)? It is almost 5 times as much as a refrigerator uses. Then why does the refrigerator use 3 times as much a year as the iron?

**5.** How much greater is the per cent of homes having radios in use than washers?

**6.** How much more wattage per hour does a coffee maker use than a TV set?

**7.** Add:

| | |
|---|---|
| 45,775,000 | 2,126,000 |
| 39,397,000 | 845,000 |
| 39,272,000 | 1,674,000 |

**8.** Subtract:

| | |
|---|---|
| 23,677,800 | 1,480,000 |
| 14,873,500 | 658,000 |

## The Cost of Phone Service

The telephone is a convenience for all people, but it is a necessity for many. In any event, everyone should know how much the telephone costs and how a bill is figured so that records can be kept straight.

**1.** Here are some rates that are probably close to what you pay for phone service where you live:

| Business rates (per month): | | | |
|---|---|---|---|
| 1-Party Business | 2-Party Business | Rural Business | |
| $8.75 | $7.25 | $5.75 | *add 10% tax* |

| Resident rates (per month): | | | | |
|---|---|---|---|---|
| Private | 2-Party | 4-Party | Rural | |
| $4.00 | $3.25 | $2.75 | $3.75 | *add 10% tax* |

Figure the monthly costs for each of these classifications. Figure the yearly costs.

**2.** The following rates are for station to station calls from Lansing, Michigan, 3 min. calls or less (federal excise tax must be added):

| | | | |
|---|---|---|---|
| Bangor, Maine | $1.60 | Miami, Florida | $1.90 |
| Chicago, Illinois | .70 | New Orleans, Louisiana | 1.70 |
| Dallas, Texas | 1.70 | Seattle, Washington | 2.20 |
| Los Angeles, California | 2.35 | St. Paul, Minnesota | 1.20 |

**a.** What would be your phone bill for the month if you had a 1-party business phone in Lansing and called Miami and Dallas once each?

**b.** What would be your bill for the month if you called New Orleans once and Seattle twice from your own 2-party residence phone?

**3.** When the U.S. population was 161,000,000, there were 28.7 phones for every 100 people. How many phones were there in the U.S. at that time? How many phones were there if the ratio stayed the same when the population reached 166,000,000?

## Practice Problems and Exercises

**1.** The first year that television receivers were put on a mass production basis was in 1947. There were 210,000 sets made that year. If the cost of making these sets was 90 million dollars, about how much did it cost to make the average set found in the home?

**2.** There has been a steady growth in the production of TV sets each year. The number of sets made in 1953 was 7 million. If it cost $1,600,000,000 to manufacture these sets, what was the average cost per unit that year?

**3.** The average hourly pay for production workers in 1941 was $.73. This rate rose in 12 years to $1.74. How much was the increase? What was the per cent of increase?

**4.** From 1941 to 1953 the average weekly earnings of production workers rose from $29.58 to $71.34. Will this give you about the same hourly increase that you got in Problem 3, figuring on a 40-hr. week? Can you think of any reason why it might be different?

**5.** Assuming that there are 166,000,000 people in the U.S. and they use a telephone 33,200 million times in a year, what is the average number of calls that each person makes?

**6.** Using 360 days for a year, what is the average number of calls that each person makes per day? (Use figures from Problem 5.)

**7.** Divide, giving your quotients as mixed numbers:

$36\overline{)880}$ $24\overline{)300}$ $144\overline{)9{,}600}$ $24\overline{)3{,}960}$

**8.** What fractional part of 96 is 80?

**9.** What per cent of 96 is 72?

**10.** Multiply 1,728 by $4\frac{3}{4}$. Divide $7\frac{1}{2}$ by $1\frac{1}{8}$.

**11.** Add:

| **a.** | **b.** |
|---|---|
| $27\frac{5}{6}$ | $40\frac{7}{8}$ |
| $18\frac{3}{4}$ | $16\frac{2}{3}$ |

**12.** Subtract:

| **a.** | **b.** |
|---|---|
| $40\frac{2}{3}$ | $66\frac{2}{5}$ |
| $19\frac{5}{6}$ | $30\frac{3}{4}$ |



## Many People Use Gasoline

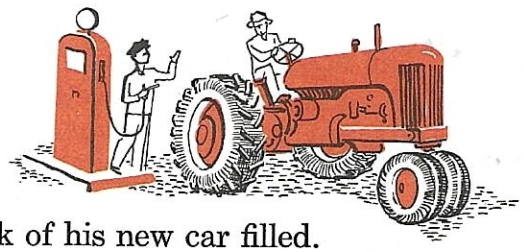
The first oil well in the United States was located in Pennsylvania. It began producing in 1859.

**1.** How many years has it been since Pennsylvania first produced oil?

**2.** There were 4,192 automobiles built in the year 1900. 50 years later, 6,665,863 were manufactured in one year. How many more were manufactured in 1950 than in 1900? How much gasoline would be used by those manufactured in 1950, traveling 10,000 mi. each, and assuming that each averaged 15 mi. per gallon?

**3.** There were 53,258,570 automobiles, trucks, and buses registered in the United States in a recent year. During the same year, 45,037,336,000 gallons of gasoline were used. Read this number. If the average tax rate per gallon was 4.83¢, what amount of gasoline tax was paid that year?

**4.** Farmer Miller is having gas put into his tractor. He pays for it by the month. Altogether he bought 83 gallons of gas last month at 28.7¢ per gallon. How much does he owe for last month?



**5.** Mr. Johnson is having the tank of his new car filled. He is trying to find out how many miles to the gallon of gas his car is getting. Can you tell him, if his speedometer read 05139.4 the last time his car was filled, and now reads 05371.7, while the meter on the gasoline pump this time reads 15.5 gallons for his full tank?

## Checking Your Understanding

Do you know why you place the decimal points in a column in addition and subtraction exercises?

How do you find where to place the point in the answer to a multiplication or division exercise in decimals?

Do these exercises:

$$\begin{array}{r} 4.84 \\ +\ 16.8 \\ \hline \end{array}$$

You can add and subtract only like units. Thus tenths are placed under tenths, ones under ones, and so on.

**1.** Add: 8, 24, 6.8, 40.5, .9, 32.65, .07.

**2.** Subtract: 42.52 from 86.8; 40.3 from 100.06.

**3.** Multiply (do you place hundredths under hundredths, tenths under tenths, and so on, here?):

$$\begin{array}{r} 4.24 \\ \times\ 6 \\ \hline \end{array}$$

6 × 4 hundredths are ? hundredths. Does this tell you at once how many decimal places there will be in the product? Why?

$$\begin{array}{r} 43.2 \\ \times\ 2.4 \\ \hline \end{array}$$

4 tenths × 2 tenths = ?. You should remember that .4 is the same as $\frac{4}{10}$. Then, $\frac{4}{10} \times \frac{2}{10}$ is hundredths. Why?

These meanings show you how to find the number of places in the answer to any multiplication exercise. By knowing the meanings you can understand this rule:

> The number of places in a product equals the sum of the number of places in the two numbers being multiplied.

**4.** Add and check:

$$\begin{array}{r} 46.8 \\ 256.47 \\ \hline \end{array} \qquad \begin{array}{r} 278.64 \\ 69.38 \\ \hline \end{array}$$

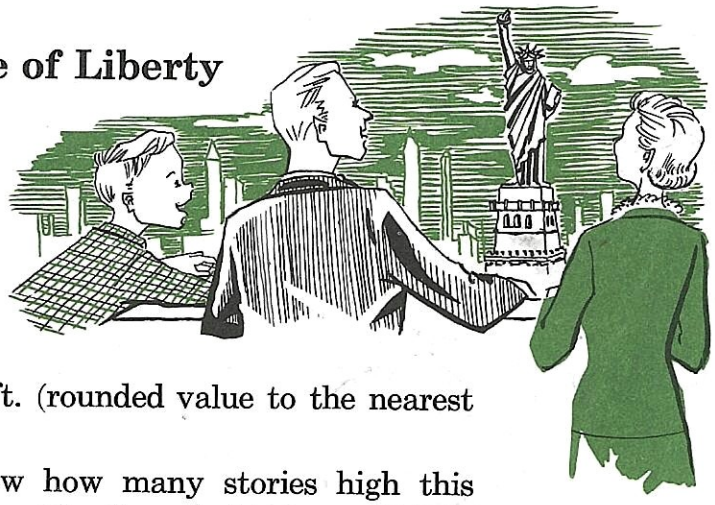
**5.** Subtract and check:

$$\begin{array}{r} 497.3 \\ 326.45 \\ \hline \end{array} \qquad \begin{array}{r} 420.06 \\ 93.49 \\ \hline \end{array}$$

**6.** Multiply:

28.37 × 6 12.67 × 4.2 968.6 × 10.5 443.4 × 2.26

## Visiting the Statue of Liberty



Harry and his family visited the Statue of Liberty in New York. A guard told Harry that the statue was 454.75 ft. above low tide in the harbor. Most people call it 455 ft. (rounded value to the nearest whole foot). Why?

Harry wanted to know how many stories high this would be if the rooms were like those in his home, 8.5 ft. from floor to floor. See if you can solve the problem:

$8.5\overline{)454.75}$

You have to divide a mixed decimal by a mixed decimal. The divisor is ones and tenths. The dividend is hundreds, tens, ones, and hundredths. Tell why.

There are two ways of doing this:

**a.** by using place values.

**b.** by changing the numbers to the same unit.

*Using Place Values:*

$8.5\overline{)454.75}$

```
    53.5
8.5)454.75
   425
    297
    255
     425
     425
```

There are not enough hundreds (4) for a value in that place when you divide 4 hundreds by 8 ones.

Use 45 tens. Why?

Tens divided by ones equals tens because you know tens × ones = tens.

Your estimated quotient figure is 5 tens. 5 tens × 8.5 ones = 42 tens and 5 ones. Why? Why is the next quotient figure (3) in ones place?

Tell why .5 is the next quotient figure.

The height of the Statue would be equal to 53.5 stories.

1. Divide by using place values:

$3.2\overline{)40.96}$ $4.5\overline{)20.25}$ $3.6\overline{)1.296}$ $5.5\overline{)30.25}$ $16.5\overline{)610.5}$

*Changing to like units:*

$3.2\overline{)40.96}$

$3.2_{\wedge}\overline{)40.9_{\wedge}6}$

$$\begin{array}{r} 12.8 \\ 3.2_{\wedge}\overline{)40.9_{\wedge}6} \\ \underline{32} \\ 89 \\ \underline{64} \\ 256 \\ \underline{256} \end{array}$$

You can change 3 ones and 2 tenths to tenths: 3.2 = 32 tenths. Why?

You must also change the dividend to tenths: 40.96 = 409.6 tenths. Why?

You place a mark called the caret ($_{\wedge}$) in each number to show the unit you are using (tenths). The decimal point is left to tell what the old units were. Your division has become 409.6 ÷ 32, both expressed in the same unit.

*Since the units are alike, you divide as you would when a mixed number is divided by a whole number.*

The caret is like a new decimal point for the new units.

The decimal point in the quotient is placed above the caret in the dividend. Why?

Another way to look at the meaning of this process is to consider the division as a fraction form (comparing two values). A division like 8 ft. ÷ 3 ft. can be written as the fraction $\frac{8 \text{ ft.}}{3 \text{ ft.}} = \frac{8}{3} = 2\frac{2}{3}$, the ratio idea.

Then, you know that $\frac{8 \text{ ft.}}{3 \text{ ft.}} = \frac{96}{36} = 36\overline{)96} = ?$ where the values are changed to higher terms.

Using this process, the above division could be written:

$$\frac{40.96}{3.2} = \frac{40.9_{\wedge}6 \text{ tenths}}{3.2_{\wedge} \text{ tenths}}$$

or $3.2_{\wedge}\overline{)40.9_{\wedge}6}$

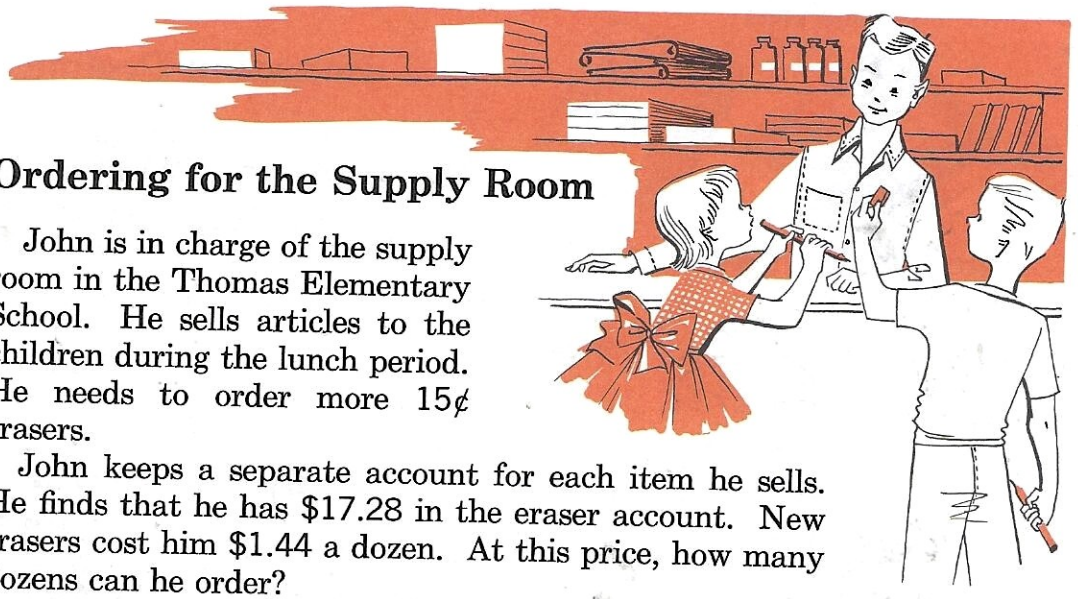
The caret tells the units in the change and the decimal point tells the old units.

The 6 in $40.9_{\wedge}6$ is now 6 tenths of a *tenth*. In 40.96 it was 6 hundredths of *one*. Why do we want like units?

You have multiplied both the numerator and denominator by 10.

**1.** Divide:

$1.2\overline{)17.28}$ $3.6\overline{)10.08}$ $5.4\overline{)38.88}$ $4.2\overline{)1.312}$



## Ordering for the Supply Room

John is in charge of the supply room in the Thomas Elementary School. He sells articles to the children during the lunch period. He needs to order more 15¢ erasers.

John keeps a separate account for each item he sells. He finds that he has $17.28 in the eraser account. New erasers cost him $1.44 a dozen. At this price, how many dozens can he order?

$\$1.44\overline{)\$17.28}$

$1.44_{\wedge}\overline{)17.28_{\wedge}}$

$$
\begin{array}{r}
12. \\
1.44_{\wedge}\overline{)17.28_{\wedge}} \\
\underline{14\,4} \\
2\,88 \\
\underline{2\,88}
\end{array}
$$

You see that you are dividing decimals with units of hundredths in each value. If you use the caret method, you change the divisor to a whole number of hundredths. Then, to have like units the dividend must also be changed to hundredths.

$100 \times 1.44 = 1.44_{\wedge}$ hundredths.
$100 \times 17.28 = 17.28_{\wedge}$ hundredths.
You now divide as whole numbers.
You should check the work to see if John can buy 12 doz. of the new erasers.

Divide, using the caret method:

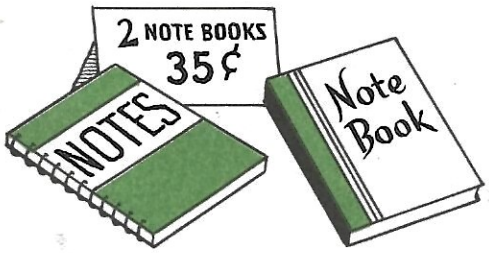
**1.** $1.26_{\wedge}\overline{)433.44_{\wedge}}$ $1.26 = 126$ hundredths or $1.26_{\wedge}$. Where will the decimal point be placed in the quotient?

**2.** $1.76\overline{)52.8}$ $1.25\overline{)8.75}$ $1.42\overline{)36.352}$ $2.16\overline{)32.832}$

**3.** $6.25\overline{)84.735}$ $.28\overline{)6.284}$ $.06\overline{)82}$ $1.24\overline{)628.5}$ $.02\overline{).05}$

**4.** Tell how you would do these divisions:

$.025_{\wedge}\overline{)6.250_{\wedge}}$ $1.728\overline{)207.36}$ $.005\overline{).025}$ $.064\overline{)409.6}$




## Rounding Decimals

John sells notebooks at 2 for 35¢, or 18¢ each. Why does he charge this price for 1 notebook?

At 2 for 35¢, the cost of 1 is found by dividing, $2\overline{)35.}$

$$2\overline{)\$.35}\quad \$.17\tfrac{1}{2}$$

But John could never collect $17\frac{1}{2}$¢. He **rounds** the amount to 18¢.

or $2\overline{)35¢} = 17.5¢$.

Sometimes the fractional parts of a quotient are kept. John would use them when he is figuring costs, but he would round to the nearest whole unit for sales. Why is the nearest whole unit a cent in $\$.17\frac{1}{2}$?

**1.** What is the nearest cent in the quotients of these divisions?

$4\overline{)\$.17}$ $5\overline{)\$.24}$ $8\overline{)\$.50}$ $6\overline{)\$.15}$ $10\overline{)\$.34}$ $3\overline{)\$2.45}$

**2.** Is there a reason why it is customary to *round* money values in cents and some measurements in one-half fractional units to the next whole unit, as in 15.5¢ rounded to 16¢? 24.5¢ to 25¢? 484.5 mi. to 485 mi.?

**3.** Why would you round values like this:

15.2¢ to 15¢? 4.3 mi. to 4 mi.? .187 to .19? 4.185 to 4.19?

**4.** Divide, and give your answers to the nearest tenths (first carry each quotient to hundredths):

$4\overline{)27}$ $5\overline{)16.3}$ $7\overline{)4.4}$ $8\overline{)6.2}$ $1.4\overline{)200}$ $.3\overline{)4.1}$

**5.** Round these numbers to the nearest hundredth:

1.145 2.768 4.283 .978 .062 .888

**6.** Why is 62.5 a value that is an equal amount from both 63 and 62?

**7.** Can you give a rule that tells how you should round values? Will it work for rounding hundredths to tenths? for rounding thousandths to hundredths?

## Dividing by 1,000

You have studied the process of dividing a value by 10 and by 100. Do you remember how to place the decimal point in these divisions?

$10\overline{)62.8} = 6.28$. Why? $100\overline{)542.6} = 5.426$. Why?

**a.** *To divide a number by* 10, *move the decimal point* 1 *place to the left.*

*Explanation:* You know why **a** is true. When ones are divided by 10, the result is tenths; when hundredths are divided by 10, the result is thousandths; and so on.

**b.** *To divide a number by* 100, *move the decimal point* 2 *places to the left.*

*Explanation:* Write the reasons for this as for **a**.

Now, what kinds of results will you get when you divide by 1,000? Let's look at an example:

```
      4.6825
1000)4682.5000
     4000
      682 5
      600 0
       82 50
```

Check this to see if the quotient is correct.

Compare the place of the decimal point in the quotient with its place in the dividend.

Can you write a rule that can be used when dividing a number by 1,000?

*To divide a number by* 1,000, *move the decimal point* 3 *places to the left.*

If you divide ones by 1,000, what is the size of the unit in your answer?

Why does dividing tens by 1,000 give hundredths?

**1.** Where is the decimal point in the value 5,280 ft.? What happens to the decimal point if you divide this by 1,000?

**2.** Divide these numbers by 1,000:

| | | | | | |
|---|---|---|---|---|---|
| 4,978 | 4.560 | 465 | 320 | 8 | 26 |
| 52,000 | 46,789 | 184.5 | 127.24 | 7.6 | .36 |

## Practice for Skill

**1.** Add:

| a. | b. | c. | d. |
|---|---|---|---|
| 62.43 | 975.63 | 6312.47 | .088 |
| 175.61 | 142.09 | 409.06 | .78 |
| 32.00 | 38.6 | 1.67 | .496 |
| 2680.1 | 1800.45 | 27.80 | .09 |
| 342.46 | 3709.07 | 1482.00 | .555 |

**2.** $43.62 + 9873.06 + 121.04 + .03 + 417.27.$

**3.** Subtract:

| a. | b. | c. | d. |
|---|---|---|---|
| 276.94 | 9746.53 | 420.00 | 5967.73 |
| 152.73 | 857.96 | 16.87 | 4858.93 |

**4.** $636.42 - 589.56 = ?$ $1673.1 - 946.86 = ?$

**5.** Multiply:

| a. | b. | c. | d. |
|---|---|---|---|
| 38.6 | 976.2 | 1375.06 | 49.402 |
| 17.3 | 43.2 | 27.38 | 14 |

**6.** $6.2 \times 3058.8$ $25.6 \times 200.8$ $24.32 \times 456.8$

| a. | b. | c. | d. |
|---|---|---|---|
| **7.** $26.5\overline{)397.5}$ | $4.38\overline{)96.36}$ | $.3\overline{)4.302}$ | $1.25\overline{)37.5}$ |
| **8.** $2.51\overline{)28.363}$ | $32.4\overline{)29.16}$ | $.97\overline{)4.656}$ | $7.6\overline{).988}$ |

**9.** How many steps, each .8 ft. high, would it take to reach the top floor of an office building that is 132.32 ft. above the ground floor?

**10.** How many steps, each .6 ft. high, would it take to reach from the heel to the top of the head of the Statue of Liberty, a distance of 111.5 ft.?

**11.** Harry's figures show that the family car will average 15.6 mi. per gallon on highway driving. If they use 40.8 gallons in 2 days, what is the approximate distance traveled?

## General Review

**1.** In preparing for a gasoline consumption race, a driver checked his distances and times. He found that it took him 1 hr. and 17 min. to get to his first check point, 3 hr. 34 min. and 23 sec. to get to the second, 2 hr. 28 min. and 51 sec. to get to the third, 5 hr. 12 min. and 47 sec. to get to the fourth, and 2 hr. 55 min. and 29 sec. to get to the last one. What was his total time for the entire run?

**2.** Mary went shopping at the supermarket. She bought a roast of beef that weighed 2 lb. 10 oz. How much should the roast cost at 75¢ a pound?

**3.** Which has the lower cost per pound, a 3 lb. 8 oz. ham for $2.59, or a 5 lb. 4 oz. ham for $3.78?

**4.** Mr. Adams made the following deposits in his savings account during the first week of July: Monday, $16.35; Tuesday, $19.20; Wednesday, $22.60; Thursday, $23.10; Friday, $12.35. What was the total for the week?

**5.** Jerry wanted an electric train set. He learned that it would cost $69.95. How much would he have to earn per week on his paper route in order to buy the train at the end of 21 weeks?

**6.** Tom found that he could walk a half mile in 10 min. At the same rate, how far could he walk in 2 hr.?

**7.** If a manufacturing plant can produce 2,650 units of materials in 25 days, what is the average per day?

**8.** Here are some recently reported estimates of the average yearly incomes per person in some of the United States:

| | | | |
|---|---|---|---|
| Alabama | $1,012 | Illinois | $1,983 |
| California | 2,032 | Kentucky | 1,135 |
| Connecticut | 2,080 | Louisiana | 1,206 |
| Delaware | 2,260 | Minnesota | 1,491 |

Draw a bar graph of the numbers (rounded to nearest hundred).

## Unit Review

Add:

| | a. | b. | c. | d. | e. | f. |
|---|---|---|---|---|---|---|
| 1. | 6042 | 223 | 644 | 7027 | 4036 | 102 |
| | 23 | 3044 | 6593 | 574 | 678 | 4008 |
| | 642 | 34 | 56 | 4567 | 3005 | 474 |
| | 6666 | 275 | 479 | 596 | 896 | 6530 |
| | 9843 | 4687 | 7594 | 5683 | 87 | 49 |

Subtract:

| | | | | | | |
|---|---|---|---|---|---|---|
| 2. | 8924 | 3006 | 4250 | 2782 | 8902 | 6101 |
| | 3215 | 227 | 338 | 1696 | 6679 | 3245 |

Multiply:

| | | | | | | |
|---|---|---|---|---|---|---|
| 3. | 1248 | 302 | 479 | 1706 | 2179 | 2365 |
| | 20 | 56 | 37 | 102 | 40 | 213 |
| 4. | 26589 | 4508 | 4003 | 2385 | 4488 | 5635 |
| | 6 | 10 | 100 | 42 | 56 | 24 |

Divide:

5. $513\overline{)21596}$ $287\overline{)3972}$ $780\overline{)22620}$ $421\overline{)31896}$

Add:

| | | | | | | |
|---|---|---|---|---|---|---|
| 6. | 24.675 | 37.9 | 23.67 | 66.84 | 3.259 | .84 |
| | 6.48 | 7.84 | 4.732 | 9.8 | 34.075 | 8.65 |
| | 77.8 | 358.9 | 44.66 | 76.08 | 9.007 | 34.08 |

Subtract:

| | | | | | | |
|---|---|---|---|---|---|---|
| 7. | 56.37 | 246.9 | 567 | 68.04 | 5.357 | 30.06 |
| | 23.57 | 98.6 | 88.7 | 9.56 | 2.49 | 7.87 |

Multiply:

| | | | | Divide: | |
|---|---|---|---|---|---|
| 8. | 32.46 | 6.48 | 236.7 | 9. $16\overline{)409.6}$ | $62.5\overline{)875}$ |
| | 6 | 1.4 | 1.25 | | |

## Unit Test

**1.** How far is it from Boston to Los Angeles if it is 478 miles from Boston to Pittsburgh, 411 miles from Pittsburgh to Chicago, 413 miles from Chicago to Kansas City, and 1352 miles from Kansas City to Los Angeles? Draw a bar graph of these distances.

**2.** John was going to ride a bus on a trip to visit his grandparents. He noticed on the schedule that he should get to their home in 4 hours and 15 minutes. He decided to see if he could figure out how many miles per hour the bus would have to average in order to make the 182.6 miles in that time. Can you figure this out for him?

**3.** Roy decided to figure how much money he might be able to make with his corn. He was sure he could get a yield of 55 bushels to the acre. If he could sell the corn for $1.48 per bushel, how much would he receive per acre?

**4.** Walt's mother and father would like to take a long trip for their family vacation. They are not sure, however, that they can afford it. They figure it would take 12 days and cost them $9.00 per night for a room, $6.75 per day for gas and oil, $12.00 per day for meals, and $5.00 per day for miscellaneous.

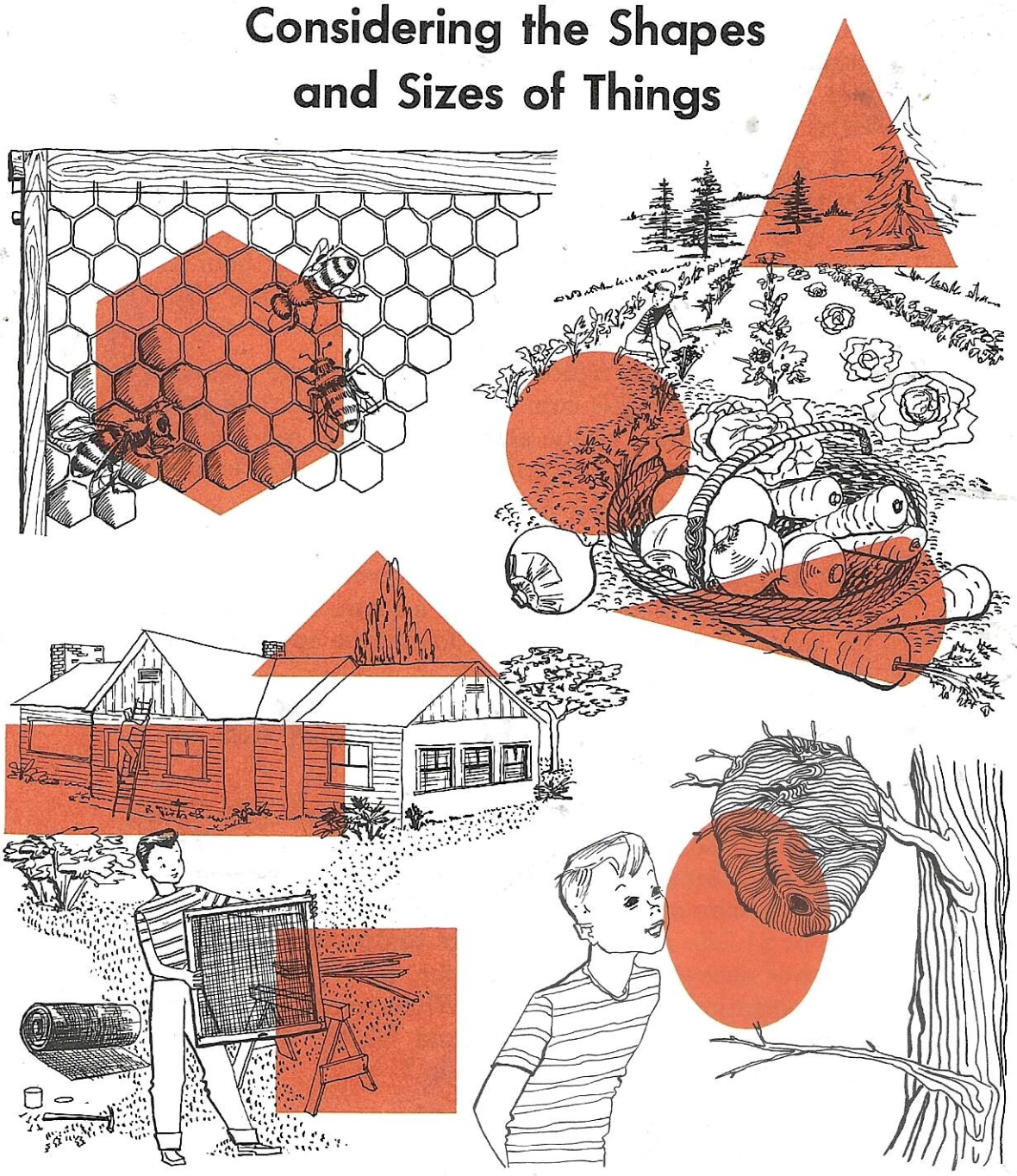
**a.** How much would the whole trip cost them?

**b.** Can they afford to take this trip if they have $300.00 to spend?

**5.** The Jones family is building a new house. Mr. Jones and his two sons are going to do much of the work. They must have some one dig the hole for the basement for them. A man offers to do the work by hand at an hourly rate of $1.25. A contractor offers to do the digging with a power shovel at $15.00 an hour, plus $10.00 for the transportation of the shovel. Which party would you employ if it would take the hand laborer 192 hr. and the power shovel 7 hr. and 20 min.?

# UNIT 11

# Considering the Shapes and Sizes of Things



Do you know what the word geometry means? In ancient times the Greeks used it to mean land measure. That is, the Greek word from which it comes had the meaning to measure land or earth.

Today, in arithmetic, the subject geometry means the study of the relations, properties, and measurement of solids, surfaces, lines, and angles. These are called geometric figures.

Do you know the names of all the different shaped geometric figures that you see in the picture?

Where do you suppose man got his ideas for the different shapes he uses? Can you name some from nature?

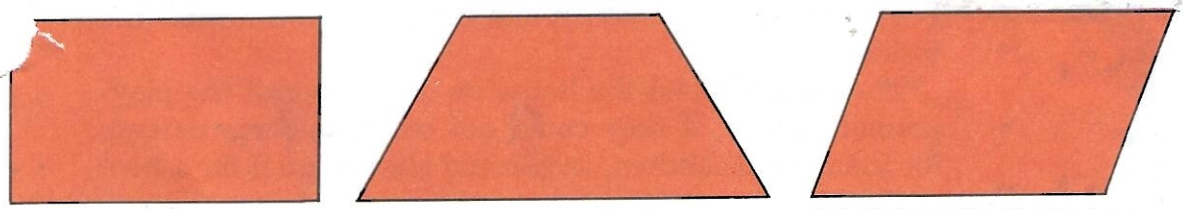
Could such things as caves, trees, the sun and moon, and many other things have given ideas to the human mind about shapes and sizes for practical uses? How?

Do you use shapes and sizes in the workshop? in cooking and sewing?

What is a rectangle? a triangle?

Do you know what it means for lines to be parallel?

Where are parallel lines used in these figures?



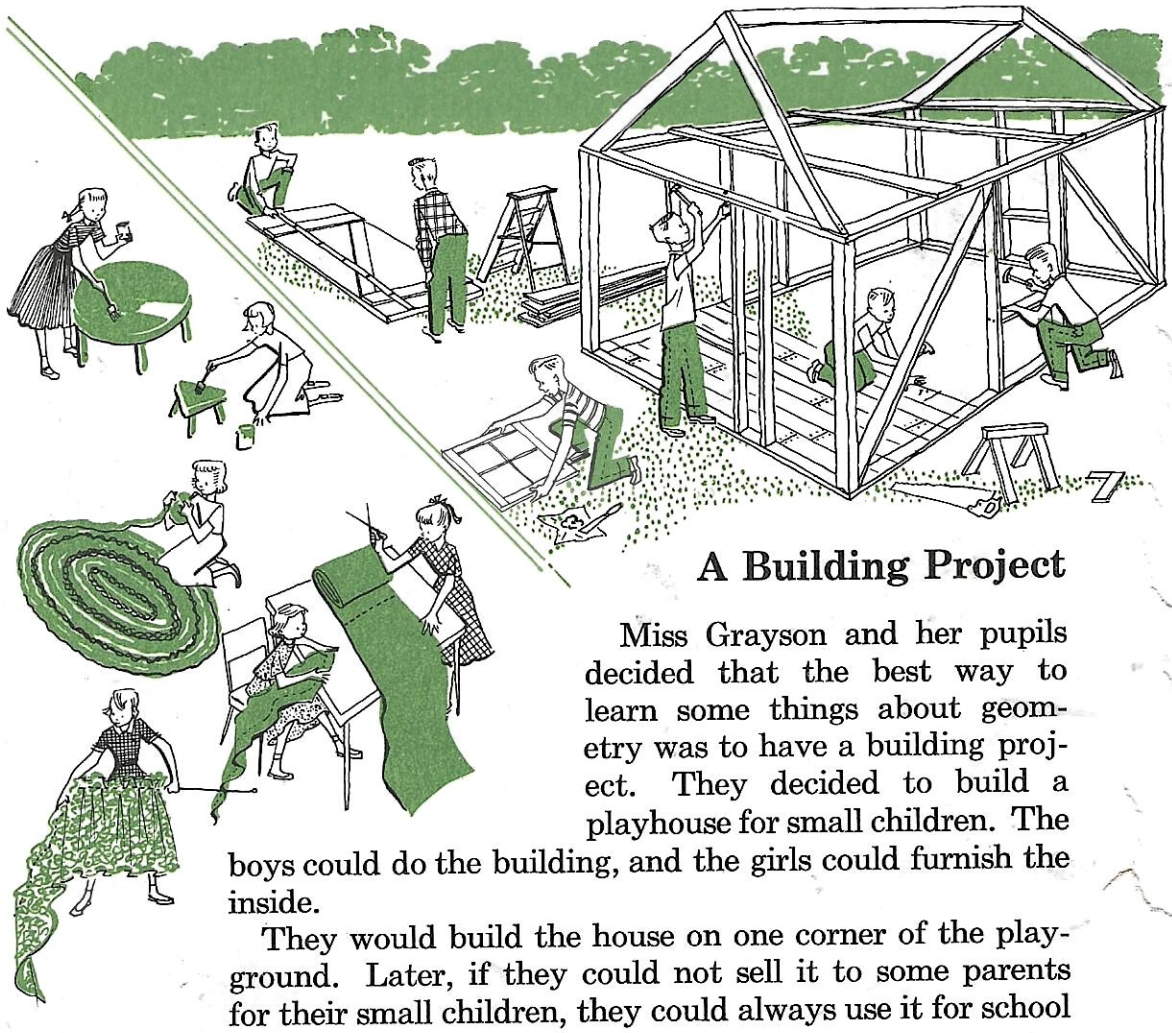
RECTANGLE TRAPEZOID PARALLELOGRAM

These are all four-sided figures. What is the name of a three-sided figure?

Does a circle have sides? What measurements can you make in a circle?

What is meant by the statement that some geometric figures enclose space?

Look up the meaning of the following words: radius, diameter, and perimeter.



## A Building Project

Miss Grayson and her pupils decided that the best way to learn some things about geometry was to have a building project. They decided to build a playhouse for small children. The boys could do the building, and the girls could furnish the inside.

They would build the house on one corner of the playground. Later, if they could not sell it to some parents for their small children, they could always use it for school activities.

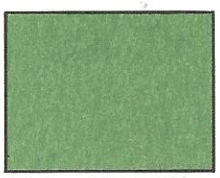
The boys would have to draw the plans first. Then, the girls would have to find the sizes of the curtains and other things that would be needed.

What are some of the shapes used in constructing a building? Where will you find these different shaped figures in the different parts of a building?

What are the shapes of an Eskimo's igloo and an Indian's tepee? Would the children build a house like either of these?

## The Plans for the Building

The boys could see that there were two basic shapes or geometric forms that would make up the building. The main one was the rectangle. They found that there are five important things to know about a rectangle.



1. It has length and width.
2. It is made up of 4 straight lines.
3. Its opposite sides are equal.
4. Its opposite sides are parallel.
5. It has 4 square corners.

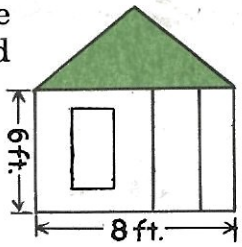
Do these same things also apply to a square? Why is a square a particular kind of rectangle?

The other basic form which the boys would use is the **triangle**. What are some important things about a triangle that they should know?



They decided that two walls of the building would be 10 ft. long and 6 ft. high. The other walls would be 8 ft. long and 6 ft. high. Then there would be 1 door and 1 window in the front wall and 1 window in each of the other walls. They would make a door 30 in. × 6 ft. and windows that were 2 ft. × 4 ft.

This is a sketch of the front of the building. What different geometric forms do you see from this view?



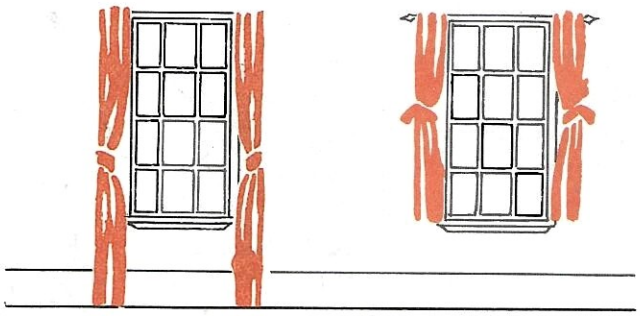

Instead of just making a free-hand sketch, the boys could have made a scale drawing. They could draw a line 1 in. long to represent a definite measurement such as 4 ft. This serves as a scale for drawing all of the lines.

1. If the boys were to use the scale 1 in. = 4 ft., how long a line must they draw to represent 10 ft.? 8 ft.? 6 ft.? Use this scale to draw the side of the building.

2. Draw a rectangle to represent a floor 12 ft. × 16 ft., using the scale 1 in. = 4 ft. This is a ratio of 1 to 48. Why?

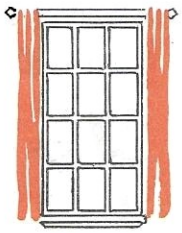
## Planning Window Curtains

The girls knew that there were 4 windows and that each one was to be 2 ft. $\times$ 4 ft. They were not quite sure how much material they would need. They could have their curtains come to the bottom of each window, or they could have them hang all the way to the floor.



They first went to a store to see what kinds of materials were available and what the prices were before making their final decision.

They found that they could buy acetate at \$.42 per yard, and nylon marquisette at \$.77 per yard. They decided to buy the acetate and to make the curtains just long enough to cover the windows. Since the windows would be 4 ft. long, they would need $\frac{4}{3}$ or $1\frac{1}{3}$ yd. for each side of a window. Why?



**1.** If each window takes $1\frac{1}{3}$ yd., how much would they need for 4 windows?

**2.** How much would the curtains cost at \$.42 per yard?

**3.** What would have been the cost for \$.77 material?

**4.** If the girls had decided to have the curtains reach the floor, their length would have been 5 ft. How many yards is 5 ft.? How much would they have needed for the 4 windows? What would have been the cost at 42¢ a yard?

**5.** What would $13\frac{1}{3}$ yards of curtain material cost at 84¢ a yard?

**6.** How many yards of material, priced at 64¢ a yard, can be bought for \$10.24? ($64\overline{)1024}$ Why?)

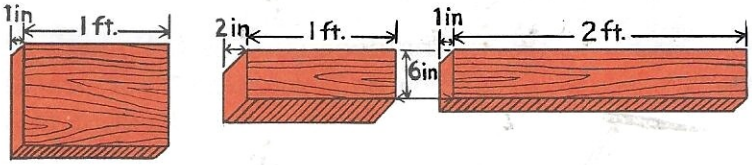
## Ordering Lumber

After the boys agreed on the size of their building, they visited a lumber yard to order what they needed.

They learned that most lumber is sold by the **board foot**. Dick asked what a board foot was. He was told that a board 1 ft. long, 1 ft. wide, and 1 in. thick was called a board foot.

Figuring board feet is like finding area, if the boards are 1 in. thick. Thus, if you find the number of square feet on the surface of boards 1 inch thick, you have the number of board feet. A board 6 ft. wide $\times$ 2 ft. long has 12 sq. ft. of surface, or 12 bd. ft.

Lumber dealers usually consider anything under an inch as being 1 in. thick.



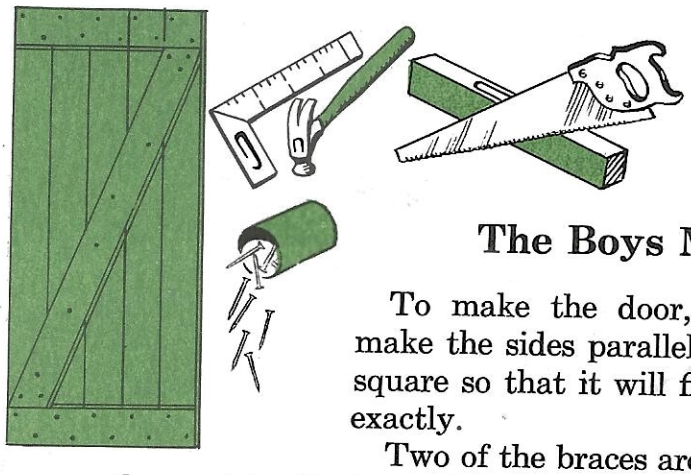

The boys saw some pieces of lumber that looked like these, each containing 1 bd. ft. Note that one piece is 2 in. thick and 6 in. wide.

The lumber dealer gave the school a special price for the materials. The bill for the cost of the different kinds of materials shows each different size of lumber. Find the total cost. (You must understand that the $\times$ sign in lumber measurements does not mean to multiply. You read $2'' \times 6'$ as $2''$ by $6'$.)

| | |
|---|---|
| 33 pieces, $1'' \times 1' \times 8'$ @ 18¢ a board foot | $47.52 |
| 1 piece, $1'' \times 6'' \times 8'$ @ 18¢ a board foot | .72 |
| 12 pieces sheathing, $1'' \times 1' \times 10'$ @ 13¢ a running ft. | 15.60 |
| 26 pieces, $2'' \times 4'' \times 8'$ @ 10¢ a running foot | 20.80 |
| 6 pieces, $2'' \times 4'' \times 10'$ @ 10¢ a running foot | 6.00 |
| 12 pieces, $2'' \times 4'' \times 6'$ @ 10¢ a running foot | 7.20 |
| 9 pieces sheet plywood, $4' \times 8'$ @ 15¢ a sq. ft. | 43.20 |
| $2\frac{1}{2}$ pieces plywood flooring, $4' \times 8'$ @ $12.80 a sheet | 32.00 |

How many board feet are there in 33 pieces, 1 ft. by 8 ft. and 1 in. thick? Is the bill correct for this?

*Figuring Board Feet*



## The Boys Make a Door

To make the door, the boys must make the sides parallel and the corners square so that it will fit into the frame exactly.

Two of the braces are parallel to each other and to the top and bottom of the door. The third brace, however, must go from one corner of the bottom brace to the other corner of the top brace to keep the door from sagging.



The door is to be 6 ft. high and 30 in. wide and made of boards 6 in. wide. For braces they also need 2 pieces of clear lumber, 1 in. thick, 6 in. wide, and $2\frac{1}{2}$ ft. long, and 1 piece 6 in. wide and 6 ft. long. Clear lumber, with no knots, will cost 24¢ a board foot.

**1.** Find the number of board feet needed for the door. You can figure this way:

5 pieces $6' \times \frac{1}{2}' = 15$ bd. ft.
2 pieces $2\frac{1}{2}' \times \frac{1}{2}' =$ ? bd. ft.
1 piece $6' \times \frac{1}{2}' =$ ? bd. ft.

If the board is 1 in. thick, the number of board feet will be the same value as its area formed by multiplying the number of units in length by the number of like units in width. This gives square units of area, or board feet if the units are in feet. Of course, you must multiply by the number of boards.

**2.** Find the cost of the materials for the door at 24¢ a board foot.

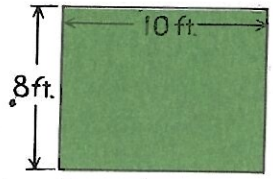
**3.** How many board feet are there in the following order of lumber?

6 pieces, $1'' \times 6'' \times 12'$
4 pieces, $1'' \times 3'' \times 8'$
12 pieces, $1'' \times 4'' \times 6'$

## How Much Trim Is Needed?

After the rough building was completed, the boys noticed that trim was needed where the walls and floor met.

They knew that the distance around a rectangle is called its **perimeter**. They could easily see that the perimeter of this room could be found by adding the four sides, 10 ft. + 8 ft. + 10 ft. + 8 ft. = 36 ft. This is the *direct method* of finding perimeters.




Ben thought it would be easier to multiply since some of the measurements were equal. They tried it just to see: Two 10's + two 8's = 36. Do you agree that Ben's idea is a good one?

Bobby suggested that they just add 2 sides before multiplying and thus save a step. $10 + 8 = 18$, and $2 \times 18 = 36$. Is Bobby's suggestion a better one?

Which one of the 3 ways of finding perimeter do you prefer?

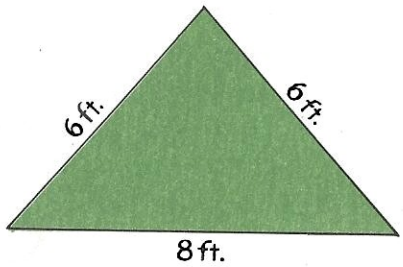
Bobby's method is called the *indirect method.*

**1.** How much molding would it take for a rectangular living room that was 18 ft. long and 14 feet wide?

**2.** This same living room had six windows. They were each $2\frac{1}{2}$ ft. wide and $4\frac{1}{2}$ ft. long. How many feet of trim would be needed to go around all six windows?

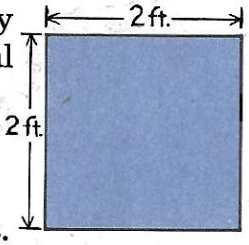
**3.** The two doorways in the room also needed trim. The pieces would need to be 6 ft. 9 in. and 3 ft. 4 in. long. How much total trim would be needed (none is needed on the bottom of a doorway)?

**4.** The front and back of the house had 2 triangles that needed trim. They were 8 ft. across and had sides 6 ft. long. How much trim would be needed for these?

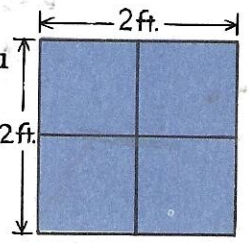


## Studying the Square

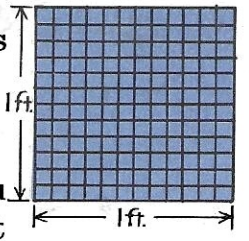
When Miss Grayson's pupils studied the square, they decided it was like a rectangle except that it has 4 equal sides.




If they wanted to find the **perimeter** of a square like this one, they could measure each side and find the sum. Thus, 2 ft. + 2 ft. + 2 ft. + 2 ft. = 8 ft. This is a *direct measurement.* So, they decided that the *indirect method* for finding perimeter was easier, especially with the square. Since all 4 sides are equal, simply multiply one side by 4. Thus, $4 \times 2$ ft. = 8 ft.




To find the **area** of a square by the direct method, you can use the same method as with the rectangle: You count the number of square units, as in this figure. But by the indirect method, the area of this square is $2 \times 2 = 4$, or 4 sq. ft.



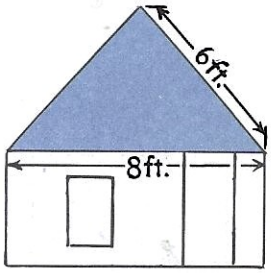

**1.** How many square inches in a square foot? This is easy when you use the indirect method. The square is 12 in. in length and 12 in. in width. When you multiply the number of inches in its length by the number in its width, you have $12 \times 12 = 144$. Thus, the area of the square foot is 144 square inches.

**2.** Find the number of square feet in a square yard.

**3.** How many square feet in a square rod (1 rd. = $16\frac{1}{2}$ ft.)?

**4.** How many square feet in the grass rug that the girls wanted and which measured 6 ft. $\times$ 9 ft.?

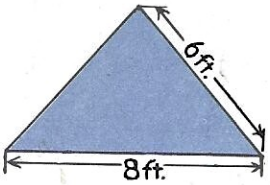
**5.** What is the area of the floor space in a square room 16 ft. $\times$ 16 ft.?



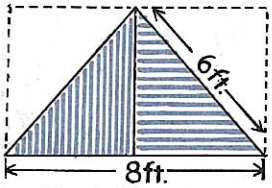

## Studying the Triangle

When the boys needed trim for the gables of their house, they had to find the perimeter of a triangle. Could they use a direct or indirect method here? Why?

To paint the house, they needed the area of the gables. Would you say that the gable is $\frac{1}{2}$ of a rectangle? Why?




Notice in the diagram shown here that the height of the gable is the same as the side of the rectangle formed by the dotted lines. Notice also that each shaded half of the triangle is equal to $\frac{1}{2}$ of a separate and smaller rectangle.



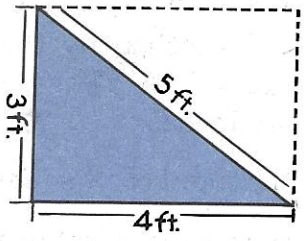

From these diagrams you can see that the area of the whole gable is exactly half of the area of the whole rectangle formed by the dotted lines.

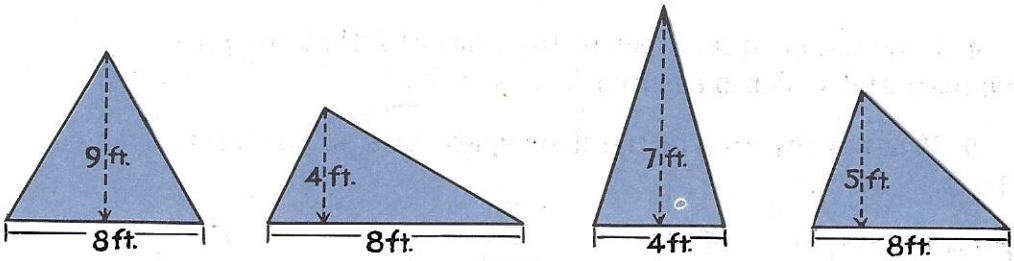
It will help you to understand if you realize that any rectangular piece of paper can be cut into halves by cutting along a line drawn from one corner to the opposite corner. (This line is called the **diagonal**.) Try it to see if the parts fit when you place one upon the other.

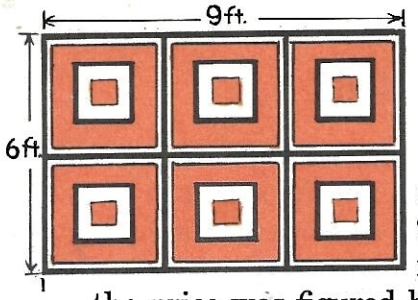
**1.** Examine these pictures, and decide if all triangles can be considered as halves of rectangles.

**2.** What is the area of the triangle which has its dimensions given?

**3.** Find the area of these triangles:

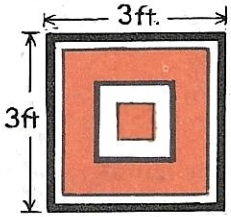





## Area of a Rectangle

The girls decided they wanted a grass rug that was 6 ft. wide and 9 ft. long. When they asked how much it would cost, they found that the price was figured by the *square yard.* They were told that a square yard was a square that measured 3 ft. on each side. They had to find how many such square units they needed for their rug. Finding how many square units are on a surface is finding its **area.**




What is the area of a rug that is 6 ft. wide and 9 ft. long?

There are two ways to find the area of a rectangle:

A. *The direct method:* Count the number of square units.

B. *The indirect method:* Multiply the number of units in one row by the number of rows.

**1.** What is the area of the boys' door (6 ft. by $2\frac{1}{2}$ ft.) in square feet? (A square foot is a square whose sides are equal to 1 ft.)

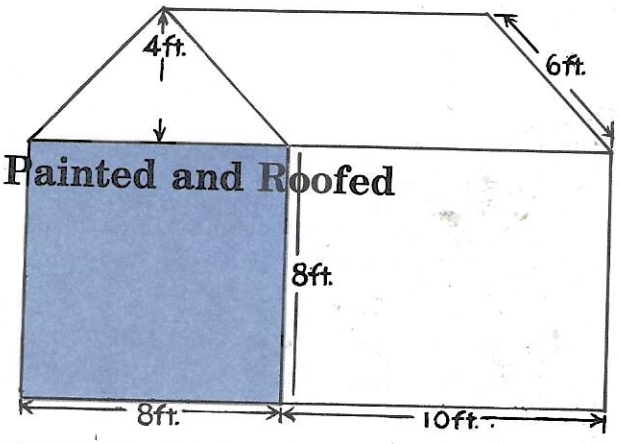
**2.** What is the area of the floor in square feet? It measures 10 ft. long and 8 ft. wide.

> To find the area of a rectangle (a square is a kind of rectangle), multiply the number of units in its length by the number of units in its width. The product gives the number of square units of area.

**3.** How much wallpaper would they need for an 8 ft. square room that had a height of 6 ft.? If there were 4 windows in the room, each 2 ft. square, and a door $2\frac{1}{2}$ ft. $\times$ 6 ft., how much less paper would be needed?

**4.** How much paper would they need for the ceiling of this square room?

## The House Must Be Painted and Roofed




A gallon of paint covers 300 square feet. How many gallons would be needed to cover this building?

2 sides 8 ft. × 8 ft.

$$\begin{array}{r} 8 \\ \times\ 8 \\ \hline 64 \\ \times\ 2 \\ \hline 128 \text{ sq. ft.} \end{array}$$

2 sides 8 ft. × 10 ft.

$$\begin{array}{r} 10 \\ \times\ 8 \\ \hline 80 \\ \times\ 2 \\ \hline 160 \text{ sq. ft.} \end{array}$$

2 gables 4 ft. × 8 ft.

$$\begin{array}{r} 4 \\ \times\ 4 \\ \hline 16 \\ \times\ 2 \\ \hline 32 \text{ sq. ft.} \end{array}$$

$$\begin{array}{r} 128 \text{ sq. ft.} \\ 160 \text{ sq. ft.} \\ 32 \text{ sq. ft.} \\ \hline 320 \text{ sq. ft.} \end{array}$$

$$300\overline{)320.00} \quad 1.07 \text{ gal., one coat}$$

$$\begin{array}{r} 300 \\ \hline 20 \end{array}$$

$$\begin{array}{r} 1.07 \text{ gal.} \\ \times\ 2 \\ \hline 2.14 \text{ gal., two coats} \end{array}$$

They plan to give the building two coats, so they must buy two gal. and 1 quart. Why? If the price is $6.00 a gallon, what will the paint cost?




**1.** If the hardware store allows them 10% off, what is the cost of the paint?

**2.** Roofing is usually figured in units called squares. A square is an area 10 ft. × 10 ft., or 100 sq. ft. How many squares of roofing are needed?

**3.** The owner of the hardware store showed them two grades of roofing, one selling at $1.34 a square and the other at $1.69 a square. They bought the grade selling at $1.69 a square. How much did the roofing cost them?

## Problems

**1.** James and Jeffrey had three boards, $5\frac{1}{2}$ ft., $2\frac{1}{2}$ ft., and 3 ft. in length. What was their total length?

**2.** One board for the roof was $7\frac{1}{2}$ ft. long. They needed one that was 6 ft. long. How much must they cut off?

**3.** Jane was asked to measure the width of some boards. She reported that one kind was $3\frac{1}{4}$ in. and others were $1\frac{1}{4}$ in. wide. How much wider were the first kind than the second?

**4.** James said the roof should be $4\frac{1}{2}$ ft. high while Jeffrey wanted it 3 ft. 2 in. high. What is the difference between the two heights?

**5.** Jane said, "We're going to use boards $\frac{1}{2}$ ft. wide and 4 ft. long on each inside wall of a room 9 ft. long and 6 ft. wide." How many boards standing upright will be needed to go around the walls? How many bd. ft. of lumber do these boards represent if they are 1 in. thick?

**6.** Jeffrey cut 1 ft. 3 in. off a board $6\frac{1}{2}$ ft. long. What was the length of the board remaining?

**7.** The boys asked Jane to decide how much they would saw off if they needed 7 ft. $7\frac{1}{2}$ in., and a board was 9 ft. 3 in. long. What is the correct answer?

**8.** When James sawed the wood, he cut off 6 ft. $1\frac{1}{2}$ in. How much shorter was this length than the 7 ft. $7\frac{1}{2}$ in. that he wanted?

**9.** Tom was to use 9 boards $6\frac{3}{4}$ ft. long. How many running feet of board will he use?

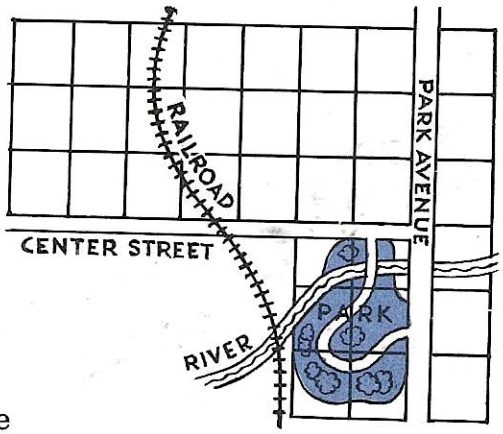
**10.** If a plank cost \$.16 for each foot of its length, what would be the cost of 6 ft. of the wood?

**11.** Paint costs \$.47 a can. Mary and Jeffrey used $3\frac{1}{4}$ cans. How much did the paint that they used cost?

**12.** The children worked $3\frac{3}{4}$ hours a day for nine days. How many hours did they work altogether?

**13.** Find the cost of $20\frac{3}{4}$ sq. ft. of roofing material at \$.39 per sq. ft.

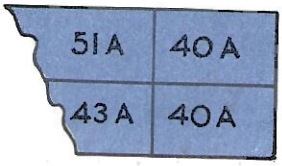
## Learning More About Area




If you live in a town or city, you know that the intersection of streets may form blocks which are square in shape.

About how many blocks are there in the section of the town shown here?

The roads in the country often divide the land into shapes that are rectangles or squares. In this diagram, you see that John Doe has two farms that are in the shape of squares. What does the 40 A. mean?




**1.** If there were 10 blocks per mile in Farm Town and Bill lived 15 blocks from school, how many miles from school did he live?

**2.** Jimmy, who also lived in Farm Town, knew that it was 32 blocks from his home to the park. How many miles did he live from the park?

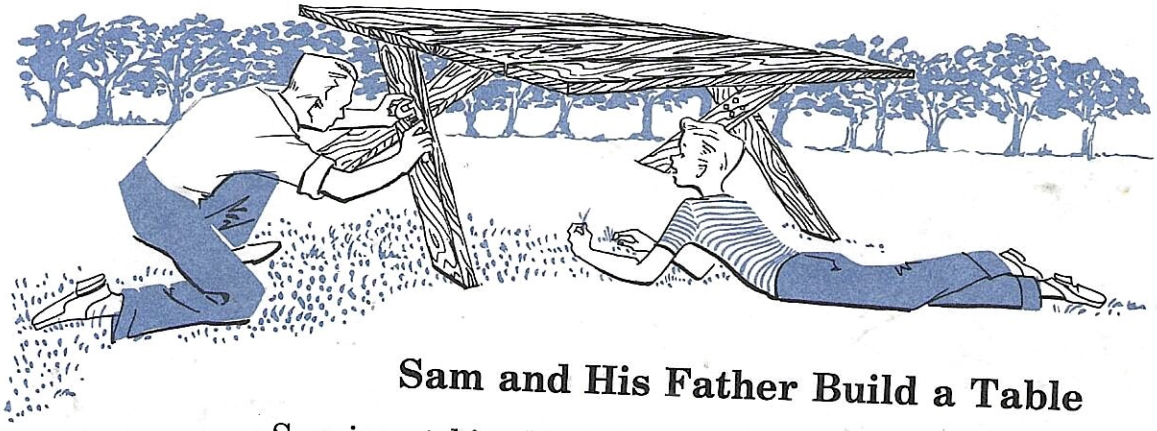
**3.** The new housing area in Bill's community is 5 miles long and 3 miles wide. How many blocks long and wide is it if there are 10 blocks per mile? 12 blocks per mile?

**4.** An **acre** is used to measure land. The size of farms is usually measured in so many acres. 1 acre = 43,560 sq. ft. There are 160 square rods in an acre. How many acres in a farm 40 rd. $\times$ 80 rd.?

**5.** A farm in the shape of a square, 1 mi. $\times$ 1 mi., is a square mile in area. Find the number of acres in a square mile of land. (1 mile = 320 rd.)

**6.** Bill had a plot of land that was 10 rods long and 8 rods wide for a garden. How many square rods did he have for the area of his garden? How many acres?

**7.** A park is 160 rods long and 150 rods wide. What is its area in square rods? in acres?

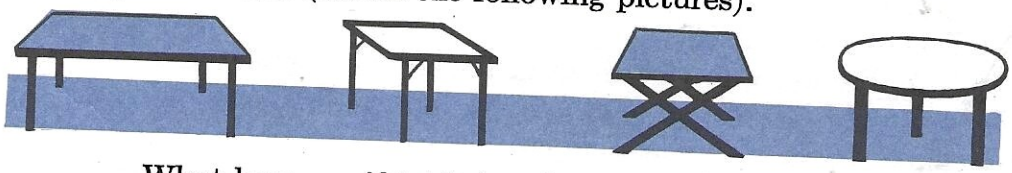


## Sam and His Father Build a Table

Sam is watching his father put the table together. Do you suppose he is telling him what makes the table sturdy?

What geometric figures do you see in the picture?

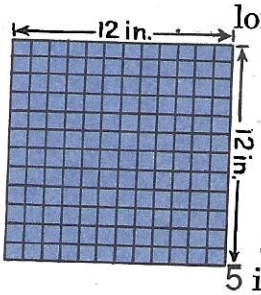
Will legs forming triangles make a table stand more steadily than legs forming rectangles? Give reasons for your answer (notice the following pictures).



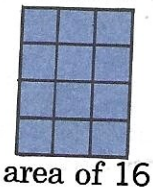
What happens if table legs become slightly loose at the joints?

**1.** If the top of a table is rectangular in shape and measures 30″ × 60″, find its area in square inches; in square feet.

**2.** What is the area of the top of a bench if it is 60″ long and 12″ wide?



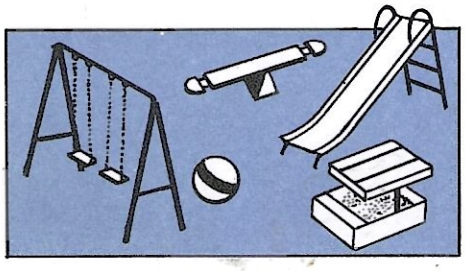

**3.** Why are there 144 square inches in one square foot?



**4.** Do you know the difference between 12 sq. in. and a 12-in. square? Which is larger? Why?

**5.** Is an area of 16 sq. in. larger or smaller than an area 5 in. square? How great is the difference?

**6.** How many square inches are there in a $1\frac{1}{2}$ ft. square?



## Problems About Perimeter

**1.** This is a sketch of a playground. What do you know about a figure having this shape?

**2.** Copy this figure on a piece of paper. Indicate which is the length and which is the width by "l" and "w."

**3.** Suppose you wanted to put a fence around a rectangular playground. How would you know how much fence to buy?

**4.** A playground is 100 yd. long and 50 yd. wide. If you add the 4 sides, what is the perimeter?

**5.** If you want to put up a small fence around a flower garden to keep out dogs and children, what would you need to know?




**6.** If the flower garden shown were 20 ft. long and 15 ft. wide, how much fence would you need to enclose the garden? Find your answer by adding the 4 sides.

**7.** Find the amount of fence you would need in Problem 6 by another method other than adding the 4 sides.

**8.** Compare your answer in Problem 6 with the one you got in Problem 7. Are they the same?

**9.** What is the distance around a room that is 12 ft. long and 10 ft. wide?

**10.** Find the perimeter of a swimming pool that is 40 ft. long and 20 ft. wide. Do not add the 4 sides. Use the shorter method.

**11.** The Barrys decided to plant a hedge around their garden. If each bush takes 1 ft. of space, how many bushes would they need to go around a garden that is $83\frac{1}{2}$ ft. long and $62\frac{1}{4}$ ft. wide?

**12.** Joe was given the job of buying rope to fence off an area in the baseball bleachers. The space is to be large enough to seat the band. If his measurements are $22\frac{1}{2}$ ft. $\times$ $14\frac{3}{4}$ ft., how much rope must he buy?

## Problems About Area

**1.** Harry broke a sheet of glass in a store window. The glass was rectangular in shape, 3 ft. × 6 ft. How many square feet were there in the glass?

**2.** The glass costs 30¢ a square foot. How much will this cost Harry?

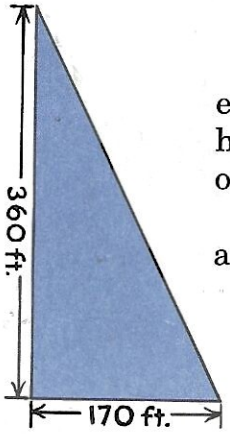
**3.** What is the area of a rectangular floor 16 ft. × 20 ft.?

**4.** Find the area of your classroom floor in square feet.

**5.** Find your classroom floor area in square yards.

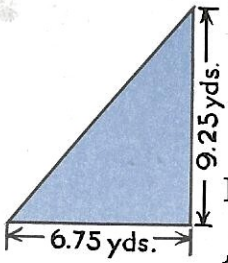
**6.** Sam makes building blocks that are 2 in. long on each side. He has a box 12 in. long, 8 in. wide, and 6 in. high. How many blocks can he place in the bottom layer of the box?

**7.** Mr. Smith's front porch is 16 ft. × 7 ft. Find its area.




**8.** If a triangular playground has one side 360 ft. long and the other 170 ft. long, how many square feet does it contain?

**9.** What does it cost to cement a tennis court 26 ft. × 78 ft. at $2.50 per square yard?




**10.** Find the area of a square-cornered triangle whose sides are 9.25 yd. by 6.75 yd.

**11.** Tom's back porch is $12\frac{1}{2}$ ft. long and $5\frac{1}{4}$ ft. wide. How many square feet of flooring will cover it?

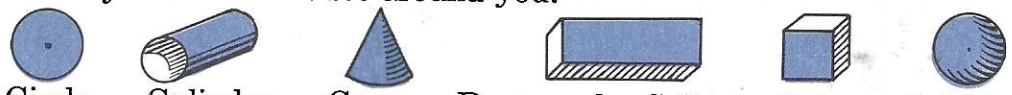
**12.** How many square feet of surface on a football field that is 100 yd. by 160 ft.? How much would it cost, at $1.25 per sq. yd., to cover the field with sod?

**13.** How many square feet of surface are on the walls of a room if the floor is 10 ft. square and the ceiling is 8 ft. high?

**14.** How many square feet of tiling would be needed to surface the floor of a room that is $31\frac{1}{2}$ ft. long and $24\frac{1}{4}$ ft. wide? How many 9-in. square tiles would be needed?

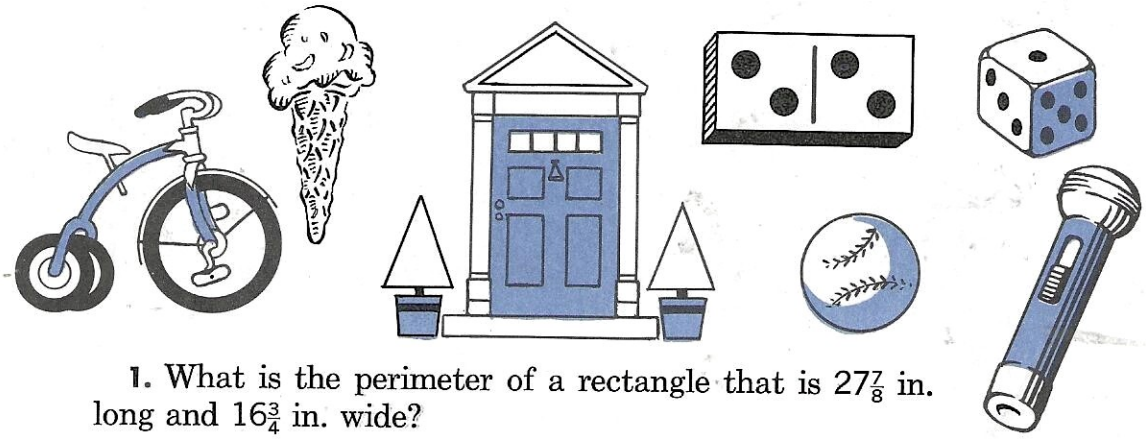
## Unit Review

Here are the names of some geometrical shapes or forms which you sometimes see around you:



Circle Cylinder Cone Rectangular Solid Cube Sphere

Tell where you might find them in nature.



**1.** What is the perimeter of a rectangle that is $27\frac{7}{8}$ in. long and $16\frac{3}{4}$ in. wide?

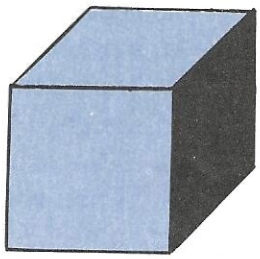
**2.** The length of one side of a square is 16 inches. What is the square's perimeter? its area?

**3.** A drawing is to be made that represents the ground floor of a building. If the scale is 1 in. = 12 ft., what length of line would be drawn to represent 42 feet?

**4.** Find the area of a triangle that has a square corner, a height of $9\frac{3}{4}$ in., and a base of $18\frac{1}{2}$ inches.

**5.** How many board feet in 6 pieces of lumber $1'' \times 6'' \times 12$ feet?

**6.** How many cubes, 1 in. on each side, could you place in the bottom layer of a box that is 18 in. long and 12 in. wide? How many layers would fill the box if it is 6 in. deep? How many cubes would it take to fill the box? (This is cubic measure or volume.)



## Unit Test

**1.** A vacant lot measures $50' \times 80'$. How many sq. ft. of lumber are needed for boards to fence the area, if we use boards standing 4′ high and allow a 5′ opening for a gate?

**2.** How many fence posts would be needed to fence the vacant lot above ($50' \times 80'$) if each is placed at a distance of ten feet from the other? (Do not count the corner posts twice.)

**3.** If a keg of nails weighs $23\frac{1}{2}$ pounds and costs $17¢$ per pound, what will be the cost of the nails?

**4.** There are 20 chairs in the boys' clubhouse which need painting. If 1 pint of paint is sufficient to give $2\frac{1}{2}$ chairs 2 coats, how much paint must be bought to give all the chairs two coats?

**5.** Ann's clubhouse has 4 square windows each measuring 3′ 4″ on a side. How many square feet of screening must the club buy for all the windows?

**6.** If the Johnsons' new square lawn is 35 feet on each side, how many pounds of seed are needed to sow this area (1 pound will seed 75 square ft.)?

**7.** Find the perimeter of the following:

**a.** a triangle with sides measuring 9 ft., 12 ft., and 15 ft.

**b.** a rectangle $15\frac{1}{4}$ in. wide and $22\frac{5}{8}$ in. long.

**c.** a 4″ square; a $16\frac{1}{4}$ in. square; a $4\frac{1}{2}$ yd. square.

**8.** How many board feet in each of these boards?

**a.** $1'' \times 8'' \times 9\frac{1}{4}'$ **b.** $3'' \times 4'' \times 7'$ **c.** $1'' \times 10'' \times 14\frac{1}{2}'$

**9.** Find the area of the following:

**a.** rectangles: $16'' \times 26''$; $7' \times 9''$; $4' \times 6''$; $16'' \times 24''$

**b.** squares: 7 ft. square; 14 ft. square; 21 ft. square.

**c.** triangles: 6 in. base and 8 in. height; 9 in. base, 12 in. height.

**10.** What does 1 in. = 10 ft. mean on a scale drawing?

**11.** What will be the size of the picture that represents a rectangle 18 ft. × 26 ft., if the scale is 1 in. = 8 ft.?

# Tables of Measure

### Length Measures

| | |
|---|---|
| 12 inches (in.) | = 1 foot (ft.) |
| 36 inches | = 1 yard (yd.) |
| 3 feet | = 1 yard |
| $16\frac{1}{2}$ feet | = 1 rod (rd.) |
| 5,280 feet | = 1 mile (mi.) |
| 320 rods | = 1 mile |

### Liquid Measures

| | |
|---|---|
| 2 cups (c.) | = 1 pint (pt.) |
| 2 pints | = 1 quart (qt.) |
| 4 quarts | = 1 gallon (gal.) |

### Dry Measures

| | |
|---|---|
| 3 teaspoons (tsp.) | = 1 tablespoon (tbs.) |
| 16 tablespoons | = 1 cup (c.) |
| 2 cups | = 1 pint (pt.) |
| 2 pints | = 1 quart (qt.) |
| 4 quarts | = 1 gallon (gal.) |
| 2 gallons | = 1 peck (pk.) |
| 4 pecks | = 1 bushel (bu.) |

### Weight Measures

| | |
|---|---|
| 16 ounces (oz.) | = 1 pound (lb.) |
| 100 pounds | = 1 hundred-weight (cwt.) |
| 2000 pounds | = 1 ton |

### Time Measures

| | | | |
|---|---|---|---|
| 60 seconds (sec.) | = 1 minute (min.) | 52 weeks | = 1 year (yr.) |
| 60 minutes | = 1 hour (hr.) | $365\frac{1}{4}$ days | = 1 year |
| 24 hours | = 1 day (da.) | 30 days | = 1 month (mo.) |
| 7 days | = 1 week (wk.) | 12 months | = 1 year |

### Volume Measures

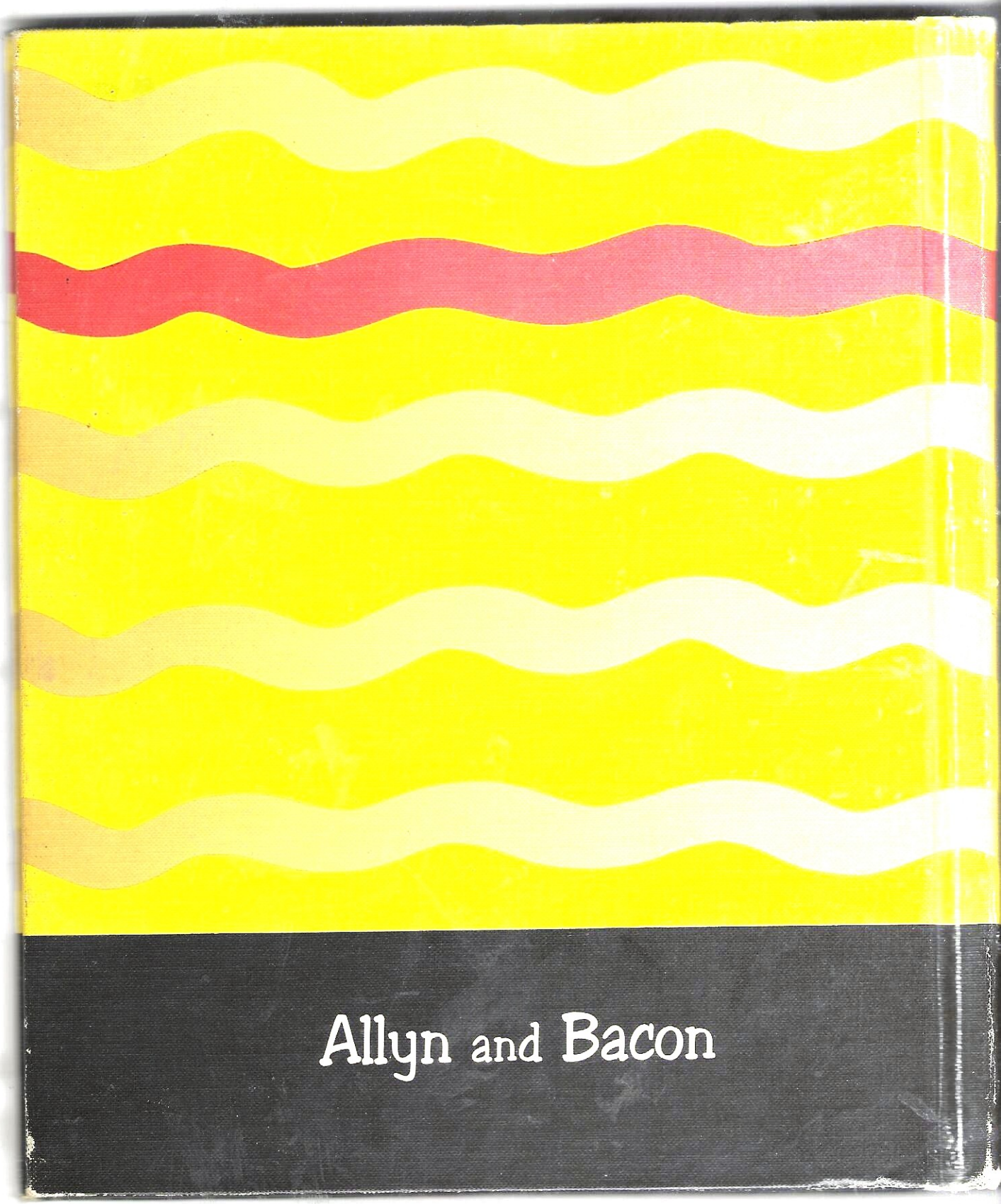
| | |
|---|---|
| 144 cubic inches (cu. in.) | = 1 cubic foot (cu. ft.) |
| 27 cubic feet | = 1 cubic yard (cu. yd.) |

### Lumber Measures

144 cubic inches—1 board foot (bd. ft.) $12'' \times 12'' \times 1''$ thick

### Square measures

| | |
|---|---|
| 144 square inches (sq. in.) | = 1 square foot (sq. ft.) |
| 9 square feet | = 1 square yard (sq. yd.) |
| $30\frac{1}{4}$ square yards | = 1 square rod (sq. rd.) |
| 160 square rods | = 1 acre (A.) |
| 640 acres | = 1 square mile (sq. mi.) |


Allyn and Bacon